

Transport Properties: Momentum Transport, Viscosity

19th February 2009

1 Introduction

Much as mass(material) is transported within fluids (gases and liquids), **linear momentum** is also associated with transport, in this case, due to **gradients in velocity**.

See Figure 35.11 for a descriptive picture of a fluid flowing between 2 fixed plates (or within a tube, as one would expect process materials to flow in a manufacturing plant, or the plumbing in your house, blood flowing in arteries and veins, etc).

Though this is discussed in the context of gases, such simple relations can hold in the liquid as well, and these connections have been borne out by experiments connected with specific solutions of the Navier-Stokes solutions.

- There is a gradient in velocity **orthogonal** to the direction of flow (for flow in x-direction, gradient along z-direction)
- We have **gradient in linear momentum** orthogonal to flow

Following the derivation of diffusion from our previous discussion, the flux of x-direction linear momentum is given by:

$$\mathbf{J}_{\text{netflux}} = -\frac{1}{3} \mathbf{v}_{\text{ave}} \tilde{N} \lambda m \left(\frac{d\mathbf{v}_x}{dz} \right)_{z=0} \quad \mathbf{J}_{\text{netflux}} = -\eta \left(\frac{d\mathbf{v}_x}{dz} \right)_{z=0} \quad (1)$$

η is the coefficient of viscosity, or viscosity. Represents a conductance of linear momentum in a fluid.

By multiplying both sides of the flux relation, we obtain the viscous drag force:

$$F_{drag} = \eta A \left(\frac{dv_x}{dz} \right)_{z=0} \quad (2)$$

In this sense, the viscosity is a measure of a fluid's resistance to flow gradients (velocity gradients).

Units of viscosity: *poise* = $10^{-1} \text{kgm}^{-1} \text{sec}^{-1}$

Gases: micropoise (μP)

Liquids: centipoise (cP)

Temperature Dependence of Viscosity

Substituting the necessary relations into the kinetic theory definition of viscosity, we obtain:

$$\eta = \sqrt{\frac{mkT}{\pi^3 \sigma^4 L}} \quad (3)$$

$$\eta \propto T^{\frac{1}{2}} \quad (4)$$

- Gases: Kinetic theory qualitatively matches experiment
- Gases: mechanism of momentum transfer is through collisions (kinetic theory); higher temperature allows greater collisions; thus greater viscous drag (fluid lamella exert force on one another)
- Liquids: Kinetic theory does not predict decrease of liquid viscosity with temperature; **Kinetic theory neglects attractive interactions/forces in liquids.**

2 Liquids: Diffusion and Viscosity

- What is the effect of a fluid's viscosity on the dynamics of a particle (**much larger than the size of the molecules of the fluid?**)
- What is the relation between the fluid viscosity and particle diffusion coefficient?

Consider a large, spherical particle in a fluid of viscosity η . The total x-direction force acting on this particle is:

$$F_{total} = F_{x,random}(t) + F_{x,friction} \quad (5)$$

$$(6)$$

The friction force is given by:

$$F_{x,friction} = -fv_x = -f \left(\frac{dx}{dt} \right) \quad (7)$$

$$f \text{ is the frictional coefficient discussed below} \quad (8)$$

The friction force for a spherical particle (larger than the molecules of the solvent) moving in a fluid of viscosity η at low Reynolds numbers (no turbulence) had been determined by George Gabriel Stokes in 1851 as a limiting solution to the more general **Navier-Stokes equations, themselves derived from considerations of mass, momentum, and energy conservation, and generalized continuity equations:**

$$\rho \frac{D\mathbf{v}}{Dt} = -\Delta p + \Delta \cdot \mathbf{T} + \mathbf{f} \quad (9)$$

The friction force is determined to be (with the frictional coefficient as $f = 6\pi\eta R$) :

$$F_{x,friction} = -6\pi\eta R v_x \quad R = \text{sphere radius} \quad (10)$$

The particle diffusion constant is then determined to be:

$$D = \frac{kT}{6\pi\eta R} \quad \text{Stokes - Einstein Equation} \quad (11)$$

- The diffusion constant depends on the first power of Temperature
- The diffusion constant varies inversely with particle size and fluid viscosity (as intuitively expected)
- For particle size and fluid molecule size of similar dimensions:

$$D = \frac{kT}{4\pi\eta R} \quad (12)$$

3 Sedimentation and Centrifugation

From the discussion of the last section, we can apply some concepts to practical separations. For the present, we consider **sedimentation** and **centrifugation**.

- Sedimentation: can be used to approach diffusion constants, molecular weights, viscosities of (macro)molecules
- Consider Figure 35.17 in Engel and Reid
- Friction Force: $F_{friction} = -fv_x$
- Gravitational Force: $F_{gravity} = mg$
- Buoyant Force: $F_{buoyant} = -m\bar{V}\rho g$
- \bar{V} = specific volume of solute
- Specific volume = change in solution volume per mass of solute (cm^3gr^{-1})
- Sedimentation velocity = terminal velocity = velocity for acceleration=0 (velocity = constant)

$$\mathbf{a} = \frac{\mathbf{F}}{m} = 0 = F_{friction} + F_{gravity} + F_{buoyant} \quad (13)$$

$$0 = -fv_{x,terminal} + mg - m\bar{V}\rho g \quad (14)$$

$$v_{x,terminal} = \frac{mg(1 - \bar{V}\rho)}{f} \quad (15)$$

$$(16)$$

- Recall f from Stokes' law

- **Sedimentation Coefficient**

$$\bar{s} = \frac{v_{x,terminal}}{g} = \frac{m(1 - \bar{V}\rho)}{f} \quad (17)$$

$$f = 6\pi\eta R \quad (18)$$

- Units of time (seconds); referred to Svedberg (s)
- **particle mass, viscosity, density, size**

If we consider that the terminal velocity is dependent on acceleration forces, such as g, we are limited (all things being equal) unless we can generate larger forces. In terms of separations, it would be ideal to have a large terminal velocity in order to speed up the separation. How do we generate larger forces? Centrifugation.

Referring to Figure 35.18 in Engell and Reid,

we can replace the gravitational acceleration by angular acceleration to arrive at the sedimentation coefficient as follows:

$$\bar{s} = \frac{v_{terminal}}{\omega^2 x} = \frac{m(1 - \bar{V}\rho)}{f} \quad (19)$$

$$\bar{s} = \frac{\left(\frac{dx(t)}{dt}\right)}{\omega^2 x(t)} = \frac{m(1 - \bar{V}\rho)}{f} \quad (20)$$

$$\omega^2 \bar{s} = \frac{\left(\frac{dx(t)}{dt}\right)}{x(t)} \quad (21)$$

$$\omega^2 \bar{s} dt = \frac{dx(t)}{x(t)} \quad (22)$$

$$\omega^2 \bar{s} t = \ln \left(\frac{x(t)}{x_o} \right) \quad (23)$$

In order to determine \bar{s} , plot $\ln(x(t)/x_o)$ versus time; slope is $\omega^2 \bar{s}$.