nag_multid_quad_monte_carlo (d01gbc)

1. Purpose

nag_multid_quad_monte_carlo (d01gbc) evaluates an approximation to the integral of a function over a hyper-rectangular region, using a Monte Carlo method. An approximate relative error estimate is also returned. This routine is suitable for low accuracy work.

2. Specification

```
#include <nag.h>
#include <nagd01.h>

void nag_multid_quad_monte_carlo(Integer ndim, double (*f)(Integer ndim,
                double x[]), Nag_MCMethod method, Nag_Start cont,
                double a[], double b[], Integer *mincls, Integer maxcls,
                double eps, double *finest, double *acc, double **comm_arr,
                NagError *fail)
```

3. Description

nag_multid_quad_monte_carlo uses an adaptive Monte Carlo method based on the algorithm described by Lautrup (1971). It is implemented for integrals of the form:

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}\ldots\int_{a_n}^{b_n} f(x_1,x_2,\ldots,x_n)\,dx_n\ldots dx_2dx_1.$$

Upon entry, unless the parameter **method** has the value **Nag_OneIteration**, the routine subdivides the integration region into a number of equal volume subregions. Inside each subregion the integral and the variance are estimated by means of pseudo-random sampling. All contributions are added together to produce an estimate for the whole integral and total variance. The variance along each co-ordinate axis is determined and the routine uses this information to increase the density and change the widths of the sub-intervals along each axis, so as to reduce the total variance. The total number of subregions is then increased by a factor of two and the program recycles for another iteration. The program stops when a desired accuracy has been reached or too many integral evaluations are needed for the next cycle.

4. Parameters

ndim

Input: the number of dimensions of the integral, n.
Constraint: **ndim** ≥ 1.

f

The function **f**, supplied by the user, must return the value of the integrand f at a given point.
The specification of **f** is:

```
double f(Integer ndim, double x[])
```

ndim
Input: the number of dimensions of the integral.

x[**ndim**]
Input: the co-ordinates of the point at which the integrand must be evaluated.

method

Input: the method to be used.
If **method** = **Nag_OneIteration**, then the function uses only one iteration of a crude Monte Carlo method with **maxcls** sample points.
If **method** = **Nag_ManyIterations**, then the function subdivides the integration region into a number of equal volume subregions.
Constraint: **method** = **Nag_OneIteration** or **Nag_ManyIterations**.

cont
Input: the continuation state of the evaluation of the integrand.
If **cont** = **Nag_Cold**, indicates that this is the first call to the routine with the current integrand and parameters **ndim**, **a** and **b**.
If **cont** = **Nag_Hot**, indicates that a previous call has been made with the same parameters **ndim**, **a** and **b** with the same integrand. Please note that **method** must not be changed.
If **cont** = **Nag_Warm**, indicates that a previous call has been made with the same parameters **ndim**, **a** and **b** but that the integrand is new. Please note that **method** must not be changed.
Constraint: **cont** = **Nag_Cold**, **Nag_Warm** or **Nag_Hot**.

a[ndim]
Input: the lower limits of integration, a_i, for $i = 1, 2, \ldots, n$.

b[ndim]
Input: the upper limits of integration, b_i, for $i = 1, 2, \ldots, n$.

mincls
Input: **mincls** must be set to the minimum number of integrand evaluations to be allowed.
Constraint: $0 \leq$ **mincls** $<$ **maxcls**.
Output: **mincls** contains the total number of integrand evaluations actually used by nag_multid_quad_monte_carlo.

maxcls
Input: the maximum number of integrand evaluations to be allowed. In the continuation case this is the number of new integrand evaluations to be allowed. These counts do not include zero integrand values.
Constraints: **maxcls** $>$ **mincls**.
maxcls $\geq 4 \times$ (**ndim** $+ 1$).

eps
Input: the relative accuracy required.
Constraint: **eps** ≥ 0.0.

finest
Output: the best estimate obtained for the integral.

acc
Output: the estimated relative accuracy of **finest**.

comm_arr
Input: if **cont** = **Nag_Warm** or **Nag_Hot**, the memory pointed to and allocated by a previous call of nag_multid_quad_monte_carlo must be unchanged.
If **cont** = **Nag_Cold** then appropriate memory is allocated internally by nag_multid_quad_monte_carlo.
Output: **comm_arr** contains information about the current sub-interval structure which could be used in later calls of nag_multid_quad_monte_carlo. In particular, **comm_arr**$[j-1]$ gives the number of sub-intervals used along the jth co-ordinate axis.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

Users are recommended to declare and initialise **fail** and set **fail.print** = TRUE for this function.

5. Error Indications and Warnings

NE_INT_ARG_LE
On entry, **mincls** must not be less than or equal to 0: **mincls** = ⟨*value*⟩.

NE_INT_ARG_LT
On entry, **ndim** must not be less than 1: **ndim** = ⟨*value*⟩.

NE_REAL_ARG_LT
On entry, **eps** must not be less than 0.0: **eps** = ⟨*value*⟩.

NE_2_INT_ARG_GE
On entry, **mincls** = ⟨*value*⟩ while **maxcls** = ⟨*value*⟩.
These parameters must satisfy **mincls** < **maxcls**.

NE_2_INT_ARG_LT
On entry, **maxcls** = ⟨*value*⟩ while **ndim** = ⟨*value*⟩.
These parameters must satisfy **maxcls** ≥ 4 × (**ndim** + 1).

NE_BAD_PARAM
On entry, parameter **method** had an illegal value.
On entry, parameter **cont** had an illegal value.

NE_QUAD_MAX_INTEGRAND_EVAL
maxcls was too small to obtain the required accuracy.
In this case nag_multid_quad_monte_carlo returns a value of **finest** with estimated relative error **acc**, but **acc** will be greater than **eps**. This error exit may be taken before **maxcls** non-zero integrand evaluations have actually occurred, if the routine calculates that the current estimates could not be improved before **maxcls** was exceeded.

NE_ALLOC_FAIL
Memory allocation failed.

6. Further Comments

The running time for nag_multid_quad_monte_carlo will usually be dominated by the time used to evaluate the integrand **f**, so the maximum time that could be used is approximately proportional to **maxcls**.

For some integrands, particularly those that are poorly behaved in a small part of the integration region, nag_multid_quad_monte_carlo may terminate with a value of **acc** which is significantly smaller than the actual relative error. This should be suspected if the returned value of **mincls** is small relative to the expected difficulty of the integral. Where this occurs, nag_multid_quad_monte_carlo should be called again, but with a higher entry value of **mincls** (e.g. twice the returned value) and the results compared with those from the previous call.

The exact values of **finest** and **acc** on return will depend (within statistical limits) on the sequence of random numbers generated within nag_multid_quad_monte_carlo by calls to nag_random_continuous_uniform (g05cac). Separate runs will produce identical answers unless the part of the program executed prior to calling nag_multid_quad_monte_carlo also calls (directly or indirectly) routines from Chapter G05, and the series of such calls differs between runs. If desired, the user may ensure the identity or difference between runs of the results returned by nag_multid_quad_monte_carlo, by calling nag_random_init_repeatable (g05cbc) or nag_random_init_nonrepeatable (g05ccc) respectively, immediately before calling nag_multid_quad_monte_carlo.

6.1. Accuracy

A relative error estimate is output through the parameter **acc**. The confidence factor is set so that the actual error should be less than **acc** 90% of the time. If a user desires a higher confidence level then a smaller value of **eps** should be used.

6.2. References

Lautrup B (1971) An Adaptive Multi-dimensional Integration Procedure *Proc. 2nd Coll. on Advanced Methods in Theoretical Physics, Marseille.*

7. See Also

nag_multid_quad_adapt (d01fcc)

8. Example

This example program calculates the integral

$$\int_0^1 \int_0^1 \int_0^1 \int_0^1 \frac{4x_1{x_3}^2 \exp(2x_1x_3)}{(1+x_2+x_4)^2} dx_1 dx_2 dx_3 dx_4 = 0.575364.$$

8.1. Program Text

```
/* nag_multid_quad_monte_carlo(d01gbc) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagd01.h>

#ifdef NAG_PROTO
static double f(Integer  ndim, double x[]);
#else
static double f();
#endif

#define NDIM 4
#define MAXCLS 20000

main()
{

  double a[4], b[4];
  Integer k, mincls;
  double finest;
  double acc, eps;
  Integer ndim = NDIM;
  Integer maxcls = MAXCLS;
  static NagError fail;
  double *comm_arr = (double *)0;
  Nag_MCMethod method;
  Nag_Start cont;

  Vprintf("d01gbc Example Program Results\n");
  for (k=0; k<4; ++k)
    {
      a[k] = 0.0;
      b[k] = 1.0;
    }
  eps = 0.01;
  mincls = 1000;
  method = Nag_ManyIterations;
  cont = Nag_Cold;
  d01gbc(ndim, f, method, cont, a, b, &mincls, maxcls, eps,
         &finest, &acc, &comm_arr, &fail);
  if (fail.code != NE_NOERROR)
    Vprintf("%s\n", fail.message);
  if (fail.code == NE_NOERROR || fail.code == NE_QUAD_MAX_INTEGRAND_EVAL)
    {
      Vprintf("Requested accuracy    = %10.2e\n",eps);
      Vprintf("Estimated value       = %10.5f\n", finest);
      Vprintf("Estimated accuracy    = %10.2e\n", acc);
      Vprintf("Number of evaluations = %5ld\n", mincls);
      exit(EXIT_SUCCESS);
    }                           /* if */
  else
    exit(EXIT_FAILURE);
}


#ifdef NAG_PROTO
static double f(Integer ndim, double x[])
#else
     static double f(ndim, x)
     Integer ndim;
```

```
     double x[];
#endif
{
  return x[0]*4.0*(x[2]*x[2])*exp(x[0]*2.0*x[2])/
    ((x[1]+1.0+x[3])*(x[1]+1.0+x[3]));
}
```

8.2. Program Data

None.

8.3. Program Results

```
d01gbc Example Program Results
Requested accuracy    =   1.00e-02
Estimated value       =    0.57554
Estimated accuracy    =   8.20e-03
Number of evaluations =  1728
```