NUMERICAL INVESTIGATION OF TURBULENT BUBBLY FLOW UNDER BREAKING WAVES

James T. Kirby, Gangfeng Ma, Morteza Derakhti, Fengyan Shi[1]

We establish a framework for describing the spatial and temporal distribution of entrained air bubbles under breaking surface waves. The computational framework is based on a large eddy simulation (LES) in 3-D together with a VOF treatment of the air-water interface. An Eulerian multiphase approach is used in order to track the entrained bubbles as a number density distribution. Examples of numerical results for breaking periodic surf zone waves are illustrated.

Keywords: wave breaking, air bubble entrainment, surf zone turbulence

INTRODUCTION

Wave breaking in the surf zone entrains large volumes of bubbles into the water column. These bubbles are involved in intense interactions with mean flow and turbulence, producing a complex two-phase bubbly flow field. It is well known that the presence of bubbles can suppress liquid phase turbulence (Wang et al., 1987; Kataoka and Serizawa, 1989; Serizawa and Kataoka, 1990; Lopez de Bertodano et al., 1994). Other studies have also revealed that bubbles may alter the local vorticity field and consequently deform or displace the vortex structure (Sridhar and Katz, 1999; Watanabe et al., 2005). Therefore, in order to study the turbulent bubbly flow under breaking waves, it is necessary to describe the dynamics of breaking waves as two-phase (gas-liquid) flow with air bubbles of appropriate size distribution. In this paper, we developed a polydisperse two-fluid bubbly flow model for surfzone breaking waves to investigate turbulent coherent structures and their interactions with dispersed bubbles. Cases considered include periodic wave breaking in the surf zone (Ting and Nelson, 2011)

THEORY

To simulate polydisperse two-fluid flow, the dispersed bubbles are separated into NG classes or groups. Each class has a characteristic bubble diameter $d_{bi}, i = 1, 2, \cdots, NG$ and a corresponding volume fraction of $\alpha_{g,i}$. By definition, the volume fraction of all of the phases must sum to one:

$$\alpha_l + \sum_{i=1}^{NG} \alpha_{g,i} = 1 \tag{1}$$

where α_l is the volume fraction of liquid phase. The volume fraction of ith bubble group is related to the bubble number density $N_{g,i}$ as

$$\alpha_{g,i} = \frac{m_{g,i} N_{g,i}}{\rho_{g,i}} \tag{2}$$

where $m_{g,i}$ is the mass of ith bubble group, $N_{g,i}$ is number density of ith group bubble and $\rho_{g,i}$ is the bubble density.

The polydisperse bubbly flow model in the current paper is based on the analysis of Carrica et al. (1999), who neglected the inertia and shear stress tensors for the gas phase due to the relatively small gas density. Following Moraga et al. (2008), we neglect bubble coalescence and gas dissolution. The governing equations consist of mass conservation for the liquid phase,

$$\frac{\partial(\alpha_l \rho_l)}{\partial t} + \nabla \cdot (\alpha_l \rho_l \mathbf{u}_l) = 0 \tag{3}$$

momentum conservation for the liquid phase,

$$\begin{aligned} &\frac{\partial(\alpha_l \rho_l \mathbf{u}_l)}{\partial t} + \nabla \cdot (\alpha_l \rho_l \mathbf{u}_l \mathbf{u}_l) = -\alpha_l \nabla p + \alpha_l \rho_l \mathbf{g} \\ &+ \nabla \cdot \left[\alpha_l \mu_{eff,l} (\nabla \mathbf{u}_l + \nabla^T \mathbf{u}_l) \right] + \mathbf{M}_{gl} \end{aligned} \tag{4}$$

[1]Center for Applied Coastal Research, University of Delaware, Newark, DE 19716 USA

bubble number density equation,

$$\frac{\partial N_{g,i}}{\partial t} + \nabla \cdot (\mathbf{u}_{g,i} N_{g,i}) = B_{g,i} + S_{g,i} + D_{g,i} \tag{5}$$

and momentum conservation for bubble phase,

$$-\alpha_{g,i} \nabla p + \alpha_{g,i} \rho_{g,i} \mathbf{g} + \mathbf{M}_{lg,i} = 0 \tag{6}$$

where ρ_l is liquid density, $\mathbf{u}_l$ is liquid velocity, p is pressure which is identical in phases, $\mathbf{g}$ is gravity, $\mu_{eff,l}$ is the effective viscosity of liquid phase, $\mathbf{u}_{g,i}$ is bubble velocity and $S_{g,i}$ is the intergroup mass transfer which only accounts for bubble breakup in the current study. The bubble breakup model proposed by Martínez-Bazán et al. (1999a,b, 2010) is employed. $D_{g,i}$ is the turbulent diffusion coefficient for the ith bubble group.

$B_{g,i}$ is the ith bubble group source due to air entrainment. In this paper, we use the bubble entrainment formulation developed by Ma et al. (2011), who correlated bubble entrainment with turbulence dissipation rate ϵ. For polydisperse bubbles, the formulation is given by

$$B_{g,i} = \frac{c_b}{4\pi} (\frac{\sigma}{\rho_l})^{-1} \alpha_l \frac{f(a_i)\Delta a_i}{\sum_{i=1}^{NG} a_i^2 f(a_i)\Delta a_i} \epsilon \tag{7}$$

where c_b is bubble entrainment coefficient which has to be calibrated in the simulation. σ is surface tension, a_i is the characteristic radius of each class, Δa_i is the width of each class and $f(a_i)$ is the bubble size spectrum, suggested by Deane and Stokes (2002).

NUMERICAL RESULTS

The numerical model is employed here to investigate the turbulent flow structures and bubble entrainment under breaking waves in a simulation corresponding to experiments by Ting and Nelson (2011) . The computational domain is tilted off-vertical to match bottom boundary with the $tan\beta = 0.03$ bed slope as shown in figure 1. The domain size is taken as 15 m long, 0.3 m wide and 0.6 m high, with the beach toe located at the left boundary. The still water depth at the beach toe is 0.36 m. A cnoidal wave with wave height of 0.122 m and wave period of 2.0 s is incident from the left boundary.

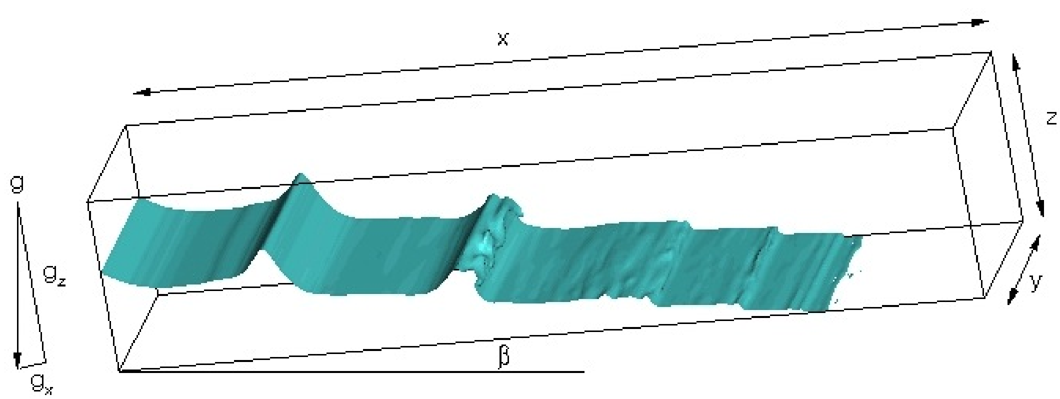

Figure 1: Computational domain and model setup for the spilling breaking wave.

Figure 2 shows the coherent vortex structures and void fraction distributions at $t = t_b + 5/8T$ and $t = t_b + 7/8T$. The coherent vortex structures are identified by the isosurface of $\lambda_2 = -5.0$, where λ_2 is the second largest eigenvalue of the tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ (Jeong and Hussain, 1995). After wave breaking, large-scale spanwise vortices will be generated near the wave crest. Behind wave crest, the vortex structures are predominantly oriented in the streamwise direction, which are well known as the obliquely descending eddies (Nadaoka et al., 1989). These vortex structures play an important role in bubble entrainment as shown in the right hand side panels of figure 2, from which we can see that bubbles are transported into the water column obliquely.

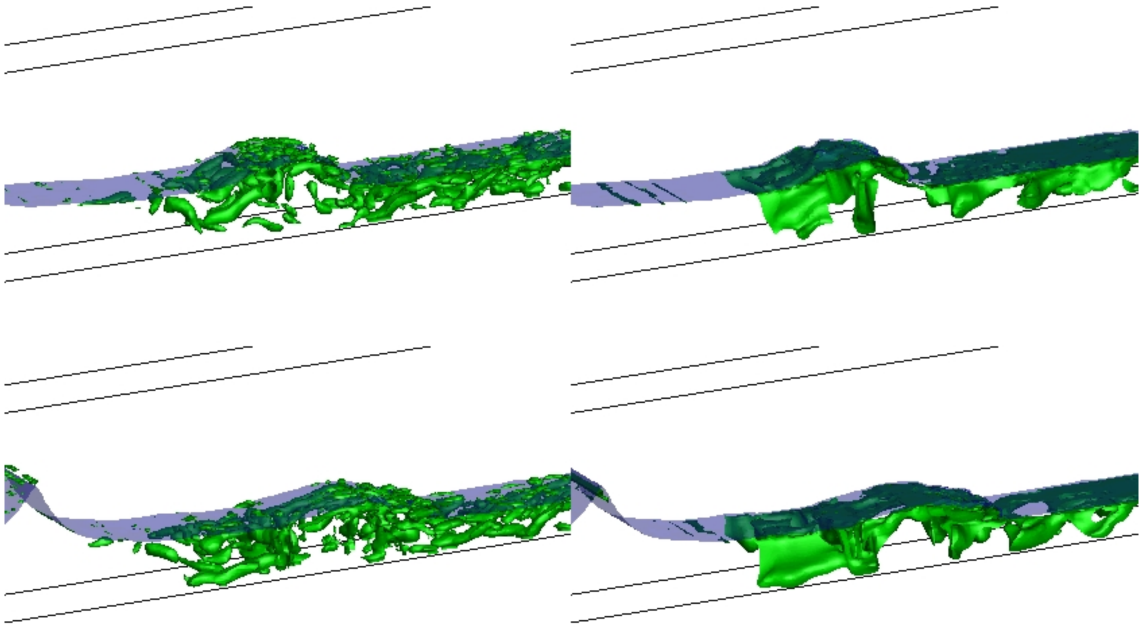

Figure 2: Coherent vortex structures (left panels) and void fraction distributions (right panels) at $t = t_b + 5/8T$ (upper panels) and $t = t_b + 7/8T$ (lower panels).

CONCLUSIONS AND FUTURE WORK

We have developed a polydisperse two-fluid bubbly flow model for surf zone breaking waves. The model accounts for dispersed bubble effects on flow field directly. The model is capable of capturing large-scale coherent structures under surf zone breaking waves, and simulating their interactions with air bubbles. Future extensions include development of a model for the representation of foam on the water surface, as well as the inclusion of sediment and chemical/biological constituents in the water column.

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