CHAPTER 133

Wave Trapping by Breakwaters

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ABSTRACT: The refraction of water waves around the heads of breakwaters can lead to large wave heights along the sheltered side of the structure. These waves are shown to be edge waves, trapped on the breakwater, based on the comparison of laboratory data and linear edge wave theory.

Introduction

Water waves experience refraction, diffraction and shoaling on a breakwater. On the seaward side of the structure, wave breaking and reflection, due to the steep sides, often dominate the wave environment along the trunk of the structure. When the angle of incidence is large (with respect to the normal to the breakwater), Mach stem reflection can occur. It is this side of the structure, which receives the most attention by engineers. On the leeward side of the breakwater, the wave climate is supposed to be reduced, and the wave field dominated by diffraction. However, the rounded head of a breakwater can cause a significant amount of wave energy to be refracted onto the leeside of the breakwater, forcing the waves to be trapped there. These trapped waves then propagate along the breakwater into the sheltered region, with wave heights far larger than expected (they can exceed the wave heights on the seaward side of the breakwater). These waves are edge waves and decrease in size only due to frictional and percolation damping along the breakwater. Fortunately the edge motion decreases rapidly away from the breakwater, so that the sheltered area is unchanged, except along the breakwater. These edge waves, with their greatest amplitudes occurring at the waterline, may be large enough to warrant their consideration in design; particularly for large structures.

This paper examines the mechanisms for wave trapping by breakwaters, showing several model predictions of the phenomenon (using both an edge wave model and a parabolic combined refraction/diffraction model) and then laboratory data are presented to show the presence of these trapped waves, verifying that they are edge wave motions.

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It has been known for a long time that the head of a breakwater is the location of wave focussing due to refraction of the waves over the submerged rounded portion of the terminus of the breakwater. Waves incident on the head of the breakwater are refracted around the structure in such a way as to create regions of very high waves on the shoulder of the structure. Palmer (1960) cites several breakwaters in the Hawaiian Islands that required repair due to this focussing. For the Nawiliwili breakwater, the extensive submerged head of the structure caused so much focussing of the waves that the waves broke and formed a jet of fluid on the lee side of the breakwater. Magoon (1984) has referred to this as the "Palmer jet". While the intent of this paper is not the discussion of the Palmer jet, it is a manifestation of the wave focussing that creates the trapped wave motion.

Smith (1986), presenting a methodology for breakwater design for the Corps of Engineers, points out that the complicated flow field in the vicinity of the breakwater head creates a design situation which is a "highly subjective and empirical process". Clearly more research into the behavior of waves in this region is important.

Theoretical Considerations

Wave-trapping on the Downwave side of Breakwaters

The simplest method for examining wave trapping by refraction around the head of the breakwater is through the use of Snell's Law. If we consider the trunk of the breakwater as a long steep beach, with parallel contours, then Snell's law applies for the refraction of the waves.

$$\frac{\sin \theta}{C} = \frac{\sin \theta_o}{C_o} \tag{1}$$

Here the subscripts denote offshore conditions. If a wave is reflected from the structure at a large angle and propagates from shallow to deep water, the wave propagation angle increases with depth. If this angle goes to 90°, then the wave is reflected from the deep water back into the shallow water (and back onto the breakwater). Due to the steepness of the breakwater, the waves, as they refract shoreward, once again reflect seaward, and the process continues. (Camfield (1982) has discussed this trapping mechanism with regard to reflective structures on straight shoreline.)

If the waves reach a 90° angle at the toe of the structure, then Snell's law reduces to

$$\sin \theta_c = \frac{C_o}{C_t} \tag{2}$$

where C_t is the wave speed at the toe of the structure and the angle, θ_c, is the critical angle. Therefore for a given depth on the breakwater, if the wave angle exceeds θ_c, then the waves will be trapped. The mechanism for inducing the waves to approach the trunk of the structure with large angles of incidence is of course the refraction at the head of the structure. See Figure 1.

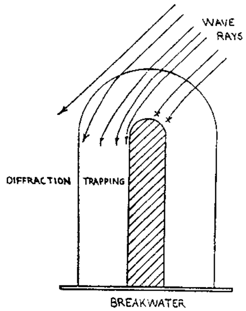

Figure 1. Schematic of Wave Trapping

It is in principle possible to determine the incident angle for wave trapping on a breakwater. Mei (1985) examines the capture of waves by circular islands, which can serve as an analog to the rounded head of a breakwater. For islands with very mild nearshore slopes, Pocinki (1950), all of the waves which cross the submerged bathymetry of the island eventually hit the island's shoreline. For islands with steeper nearshore slopes, less of the incident waves refract into the islands. This would be true for breakwaters. For waves normally incident on a breakwater, some percentage of the waves which pass the head of the structure will be captured; this percentage increasing with decreasing nearshore slope.

The trapped wave motion can be described by edge wave theory, which describes waves, which travel along a beach, rather than incident from offshore. These waves have been studied extensively by Guza (1985), who discussed the state of the art. Recently, Yeh (1986) reports laboratory verification of the edge wave theories to higher order. The dispersion relationship for a small amplitude edge wave on a planar beach of constant slope is

$$\sigma^2 = g \lambda \sin (2n+1)\beta \tag{3}$$

where σ is the angular frequency of the waves, $\sigma = 2\pi/T$, λ is the wave number of the edge wave, $\lambda = 2\pi/L$, where L is the wave length, g is the acceleration of gravity and β is the beach slope. Here n is the modal number, which describes how many zero crossings may occur in the wave profile in the offshore direction. Thus for a given breakwater slope and incident wave period the wave length of the edge waves can be determined. The free surface displacement of a zeroth mode standing edge wave can be described by an exponential function in the "offshore" (normal to the breakwater) direction (y):

$$\eta(x,y,t) = A e^{-\lambda y} \sin (\lambda x) \cos (\sigma t) \tag{4}$$

If the water depth at the toe of the structure is not too deep, then the edge waves are affected by the change in slope from the sloping structure to the bottom slope. The above dispersion relationship is no longer valid and therefore another means is necessary to find the wave length. Kirby, Dalrymple and Liu (1981) present a finite difference procedure for solving the mild-slope equation for edge waves. This procedure was used here to compute wave lengths of laboratory waves, although it was found for the depths and slopes used here that Eq. (3) is quite accurate.

The mild-slope equation for periodic waves on an arbitrary beach profile can be written as

$$p\ \eta_{yy} + p_y\ \eta_y + (\sigma^2 q - \lambda^2\ p)\ \eta = 0 \qquad (5)$$

where the subscripts denote derivatives and the water surface displacement is assumed to be

$$\hat{\eta} = \eta(y)\ \cos(\lambda x)\ \cos(\sigma t) \qquad (6)$$

and $p = C\ C_g$

$q = C/C_g$

Here C and C_g are determined from linear wave theory, based on the local depth. By introducing the variable, $\zeta = \eta y$, Kirby et al. reduced the problem to an eigenvalue problem, as $\zeta \to 0$ as $y \to 0$ and $\zeta \to 0$ as $y \to \infty$. It then becomes a matter of finding the correct value of λ for the given wave period and beach profile. Holman and Bowen (1979) solved this problem using a shooting method, which involved many iterations for each mode of the edge wave motion. Using a finite difference technique, Kirby et al. were able to obtain all edge wave modes at one time, without iteration.

The solution follows by dividing the offshore distance, x, into a number of discrete intervals, from 0 to a large number representing infinity. Expressing Eq. 5 in finite difference form (using central differences), a tri-diagonal eigenfunction equation results.

$$\{A\}\ \{\zeta_i\} = \lambda^2\ \{\zeta_i\} \qquad (7)$$

where {A} is a coefficient matrix and ζ_i are the discretized dependent variable. (This equation is slightly simpler than Kirby et al. as we have divided through by $p_i y_i^2$.) Solving the eigenvalue problem, using standard mathematics libraries (ISML), yields the eigenvalues, λ, which are the alongshore wave length of the edge waves, and the forms of the associated eigenfunction.

Wave Propagation Modelling

A refraction-diffraction model is capable of modelling the behavior of waves in the vicinity of the breakwater. In fact, the presence of the edge wave motion along the breakwater was first observed by the model, developed by Kirby and Dalrymple (1983, 1986). The parabolic model can be described as a solution to the following complex differential equation:

$$A_x - i(k-k_o)A + \frac{1}{2C_g} C_{g,x} A - \frac{i}{2C_g \sigma} (CC_g A_y)_y = 0 \qquad (8)$$

where the water surface displacement, η, is given by

$$\eta = \mathrm{Re}\ \{A(x,y)\ e^{i(k_o x - \sigma t)}\} \qquad (9)$$

and $i = \sqrt{-1}$. Here k_o is a representative wave number for the region of interest. This model is developed in finite difference form and solved by the Crank-Nicolson procedure. The results of the model are the complex amplitude, A, from which the instantaneous water surface or the transmission coefficient can be found. The reader is referred to the above papers and Kirby and Dalrymple (1984) for more details on the model, REF/DIF 1.

Laboratory Experiments

The Ocean Engineering Laboratory of the University of Delaware has a directional wave basin, with dimensions of 20m x 20m x 1.1m and a thirty element wave generator along one wall. The wave paddles, driven by Mavilor 600 servo-motors, can create waves in different directions as they are individually programmable using an HP 1000 computer.

A model breakwater with 2:3 slopes was built in the basin with a rounded head and a trunk length of 2.41m. The width of the breakwater trunk at the bottom was 2.1 m. The water depth of the basin was 0.36m. The angle of incidence of the waves was 37.8°. See figure 2. A reflecting wall was placed at the downwave end of the breakwater to prevent waves from flanking the structure, producing spurious effects, and to enhance the edge waves by reflection. This produced a partial standing edge wave pattern along the trunk of the breakwater on the downwave side.

Wave data was taken at four locations, using resistance wire gages. Gages were calibrated before and after each experiment to ensure accurate conversion of voltage data to elevation data. In front of the breakwater, the incident wave was measured at location A in figure 2. Then three wave gages were located on a transect

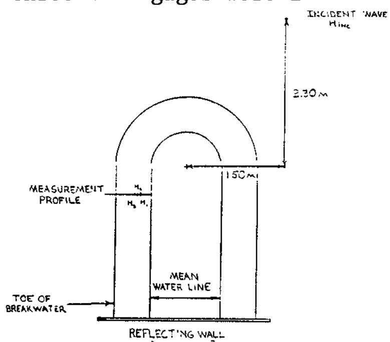

Figure 2. Schematic of Laboratory Experiment

normal to the breakwater axis, spaced 0.155m, 0.22 m, and 0.295 m from the still water line. This transect is also shown on the figure. Four separate wave periods were used for the tests. The wave heights at the wave gages were obtained for a given wave train.

Table 1 contains the wave data for the various tests, where T is the incident wave period and H_i is the incident wave height. In Table 2, the measured wave heights at the three shore-normal gages (with H_1 being the most inshore gage) are shown along with the measured spacing between the crests of the standing wave pattern along the breakwater, Lr. (This distance is one-half of the edge wavelength.)

TABLE 1. INCIDENT WAVE CHARACTERISTICS

TEST	T(s)	H_i(m)	θ(°)
234	1.00	0.068	37.8
227	1.30	0.057	37.8
212	1.35	0.091	37.8
230	1.40	0.068	37.8

TABLE 2. MEASURED WAVE DATA

Test	H_1(m)	H_2(m)	H_3(m)	L_r(m)
234	0.123	0.0860	0.0406	0.44
227	0.218	0.165	0.104	0.77
212	0.145	0.0930	0.0673	0.80
230	0.119	0.0630	0.0343	0.80

Comparing the wave heights at the innermost wave gage (H_1) for the different tests shows that there is a preferential amplification of the wave motion trapped along the breakwater. Dividing the wave height, H_1, by the incident wave heights for the four tests, we obtain the following ratios: 1.8, 3.8, 1.6, 1.7. The amplification of the waves along the trunk of the breakwater is clearly significant, with the 1.3 s. test (#227), showing almost a four-fold increase in wave height. This is thought to be due to the reflection that may occur at the breakwater head. The presence of the reflecting wall at the downwave end of the breakwater forces a partial standing wave system for the trapped waves. If the waves for test 227 are more efficiently reflected by the change in bathymetry at the head of the structure, then a more resonant situation occurs.

In Table 3, two comparisons are shown with the data and the results from the theoretical edge wave model. The measured spacing of the antinodes of the partial standing edge waves is related to λ by $\lambda = \pi / L_r$. Column 2 and 3 show the comparison of the data and the numerical model, respectively. The average error is 8.3%. The discrepancy is not likely to be attributed to wave nonlinearities, as the nonlinear terms in the dispersion relationship act to increase the wave length (Guza and Bowen, 1976). It should also be noted that for these tests, the effect of the break in slope at the toe of the breakwater is negligible. Calculating the edge wave length using the dispersion relationship for constant slope, Eq. 3, results in a maximum 2% discrepancy with that predicted with the numerical model.

TABLE 3. COMPARISON OF DATA TO NUMERICAL MODEL

TEST	L_r(m)	L_p(m)	E (%)	λ_{m1}(m^{-1})	λ_{m2}(m^{-1})	λ_p(m^{-1})
234	0.44	0.47	−6.8	7.97	7.14	6.63
227	0.77	0.81	−5.2	5.3	3.88	3.87
212	0.80	0.86	−7.5	5.44	3.92	3.64
230	0.80	0.91	−13.8	8.89	3.92	3.43

In columns 5 and 6 of Table 3, the λ determined from the laboratory data is calculated two ways. Column 5 is the wave number calculated by π/Lr. In column 6, the wave number is found by a least squares fit to the three measured wave heights for each test, found by minimizing the total error, E:

$$E = \sum_{i}^{3} (H_i - A e^{-\lambda y_i})^2$$

for A and λ. There is a discrepancy between the two measures of λ. In Column 7, the value obtained from the numerical calculation is shown. The A values found by the least squares method for the four tests, which are a measure of the wave height at the shoreline, are 0.448, 0.509, 0.326, 0.464 m in order of increasing wave period.

In figures 3 and 4, the wave heights for four tests are shown on semi-log scale. According to the edge wave theory, the zeroth mode edge wave should decay exponentially, with a slope equal to the wave number, λ. The measured data are shown with a corresponding straight line, which corresponds to the best fit line found by minimizing E, given above, for A alone, with λ given by the numerical model of Kirby et al. (Col. 7, Table 3). The agreement between the slope of the data and the straight lines in three of the tests (excepting test 230) indicates that the offshore decay of the measured wave motion is well described by an exponential behavior, as occurs with edge waves.

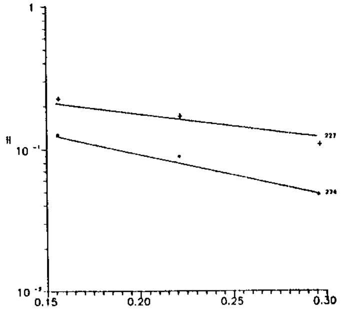

Figure 3. Wave Height Decrease with Offshore Distance for Tests 234 and 227.

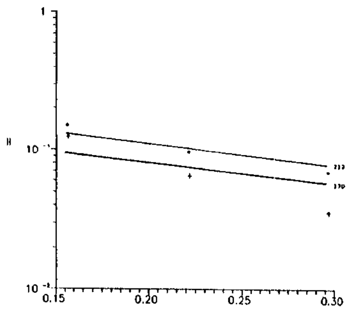

Figure 4. Wave Height Decrease with Offshore Distance for Tests 212 and 230.

In figure 5, the results of the refraction/diffraction calculation are shown. In this case, there is no reflective barrier at the end of the breakwater and thus the edge waves are progressive. For ease in modelling, two breakwaters are used. One situated on the upwave lateral boundary of the model and another on the downwave side. See Fig. 6. This permits the modelling of both the seaward and the downwave sides of the breakwater at the same time; although care must be taken to ensure that the structures are separated far enough so that the reflection from the downwave breakwater does not affect the upwave structure. Further, although the slope of the breakwater remains the same as in the physical model test, a 5 cm wall is assumed to exist along the water line, to ensure no wave breaking. The corresponding bathymetry is shown in figure 6. It should be obvious from the figure 5 that the edge wave motion becomes trapped onto the downwave side of the breakwater. (Despite appearances, the edge waves do not increase in amplitude with distance along the breakwater. They just become more prominent, as they go deeper into the diffraction zone.)

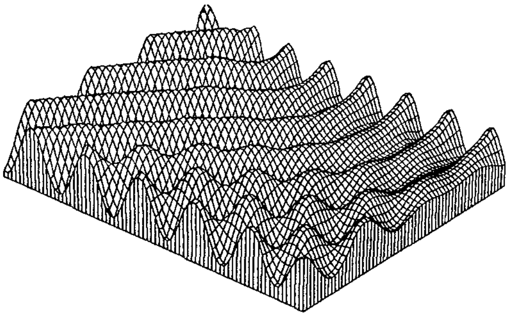

Figure 5. Instantaneous Water Surface Elevation Around Breakwaters.

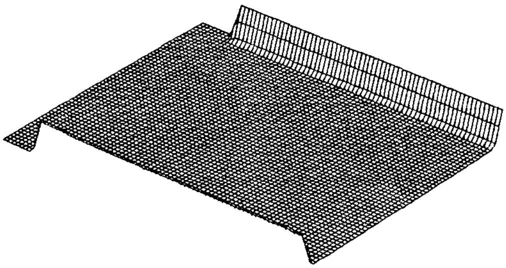

Figure 6. Bathymetry for Figure 5.

It can be presumed that should the edge wave motion on breakwaters become important, there are means to reduce their amplitudes. Clearly artificial roughness reduces their size. Another method of reducing their size is to reduce the amount of trapping that occurs. This can be done by reducing the amount of submerged head to refract the waves. Another possibility is to induce breaking on the head or to make a discontinuity in the breakwater bathymetry. In figures 7 and 8, the breakwater head is altered by adding a circular tip, which extends beyond the trunk width of the breakwater. This change reduces the amount of wave energy which can propagate around the structure.

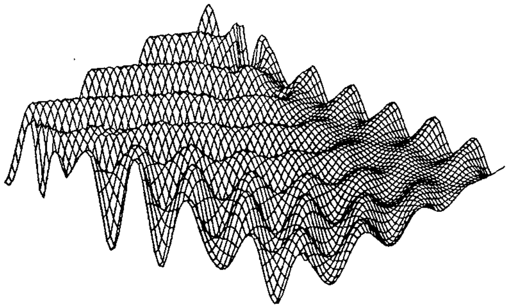

Figure 7. **Instantaneous Water Surface Elevation for Rounded Head Breakwater.**

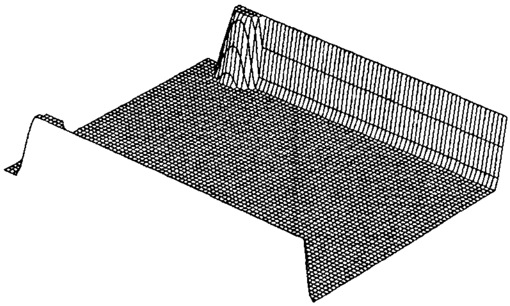

Figure 8. **Bathymetry for Figure 7.**

Conclusions

Waves, propagating around the head of a breakwater, can be channelled onto the lee side of the structure, where they will propagate, in the absence of friction, as edge waves. The laboratory data, obtained on a model breakwater, show trapped wave motion, which has the wave length and offshore dependency corresponding to edge waves.

Should this motion be of significance in the field (for large breakwaters), there are means to explore to reduce the amount of wave trapping that can occur. Further, by adding damping to the breakwater and reducing its reflectivity, the size of these waves can be reduced.

Acknowlegments

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References

Camfield, F.E., "Calculation of Trapped Reflected Waves", J. Waterway, Port, Coastal and Ocean Div., ASCE, 108, 1, 109-111, 1982.

Guza, R.T., "Laboratory Modelling of Edge Waves", in Physical Modelling in Coastal Engineering, A.A. Balkema, Rotterdam, 1985.

Guza, R.T. and A.J. Bowen, "Finite Amplitude Edge Waves", J. Marine Research, 34, 269-293.

Holman, R.A. and A.J. Bowen, "Edge Waves on Complex Beach Profiles", J. Geophys. Res., 84, 6339-6346, 1979.

Kirby, J.T. and R.A. Dalrymple, "A Parabolic Equation for the Combined Refraction-Diffusion of Stokes Waves by Mildly Varying Topography," J. Fluid Mech., Vol. 136, Nov., 1983.

Kirby, J.T. and R.A. Dalrymple, "Modeling Waves in Surfzones and Around Islands," Journal of the Waterway, Port, Coastal and Ocean Division, ASCE, 112, 1, 1986.

Kirby, J.T. and R.A. Dalrymple, "Verification of a Parabolic Equation for Propagation of Weakly-Nonlinear Waves," Coastal Engineering, May 1984.

Kirby, J.T., R.A. Dalrymple, P.L-F. Liu, "Modification of Edge Waves by Barred-Beach Topography," Coastal Engineering, 5, 35-41, 1981.

Magoon, O., personal communication, 19th Int. Coastal Eng. Conf., 1984.

Mei, C.C., The Applied Dynamics of the Ocean Surface Waves, McGraw-Hill

Palmer, R.Q.,"Breakwaters in the Hawaiian Islands", J. Waterways and Harbor Div., ASCE, 86, WW2, 39-67, 1960.

Pocinki, L.S., "The Application of Conformal Transformations to Ocean Wave Refraction Problems", Trans. Amer. Geophys. Un., 31, 6, 856-866, 1950.

Smith, O.P., "Cost-effective Optimization of Rubble-mound Breakwater Cross Sections", U.S. Army Coastal Engineering Research Center, T.R. CERC-86-2, February, 78pp. plus Appendices, 1986.

Yeh, H.H., "Experimental Study of Standing Edge Waves", J. Fluid Mech., 168, 291-304, 1986.