Formation of Hydro-acoustic Waves in Weakly Compressible Fluid Interacting with Viscous Weakly Compressible Seabed

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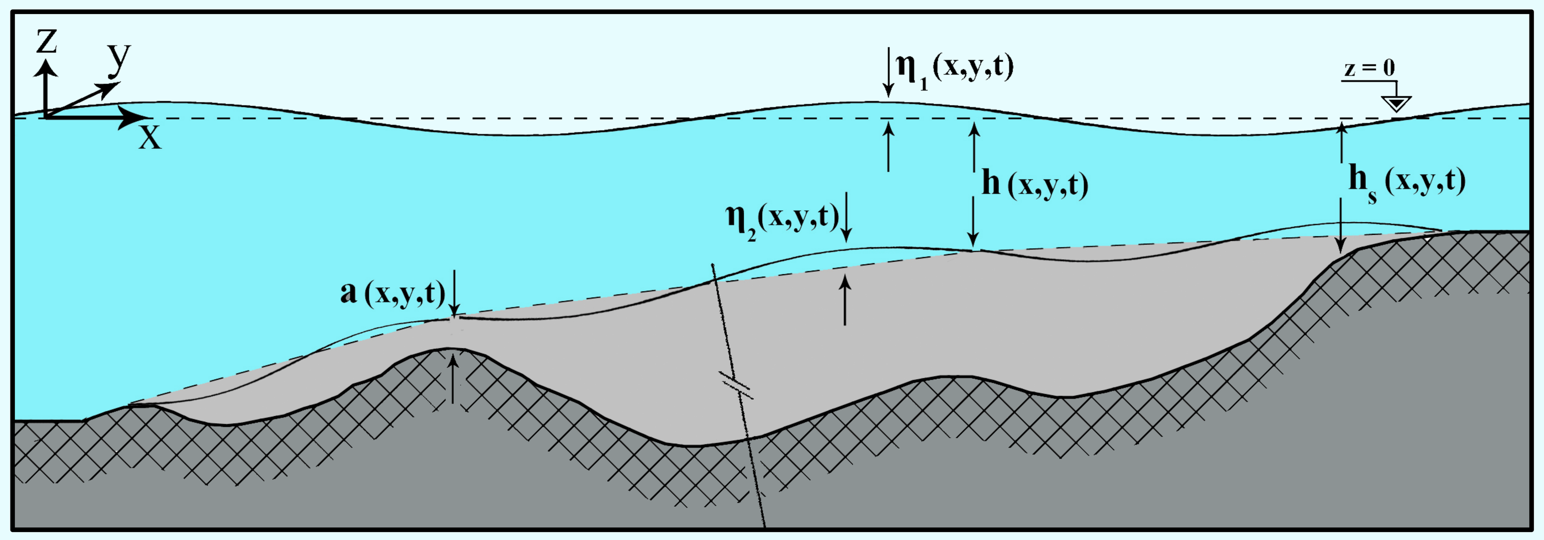
1. Objective

- **Enhancement of Tsunami Early Warning Systems (TEWS)**
  Hydro-acoustic wave detection as precursor component of tsunamis traveling at the speed of sound in water (1500 m/s) [2].
- **Study of the characteristics of hydro-acoustic waves generated by sudden sea bottom motion in a weakly compressible fluid coupled with underlying viscous weakly compressible sedimentary layer [1].**
- **Derivation of a depth integrated equation valid for varying water depth and sediment thickness .**
  Overcoming the computational difficulties of three-dimensional models and limits of available theoretical solutions [3].
- **Improving the accuracy of model by taking into account damping behavior of two layered system**
  Introducing the viscosity of fluid-like sedimentary layer.

2. Governing Equation

Weakly Compressible water.
Viscous Weakly Compressible Sediment.

$$\begin{cases} \Phi_{tt} - c^2\nabla^2\Phi = 0 \; ; & -h+\eta_2(x,y,t) \le z \le \eta_1(x,y,t) \\ Q_{tt} - c_s^2\nabla^2 Q - 2\upsilon_s(\nabla^2 Q)_t = 0 \; ; & -h_s \le z \le -h+\eta_2(x,y,t) \end{cases}$$



Schematic view of fluid domain

Boundary conditions

$$\begin{cases} \Phi_{tt} + g\,\Phi_z = 0 \; ; & z=0 \\ Q_z + \nabla_h h_s . \nabla_h Q + h_{s,t} = 0 \; ; & z=-h_s \end{cases} \quad \left| \begin{array}{l} (R-1)\,g\,\eta_2 = \Phi_t - R\,Q_t \\ W_w = W_s = (-h+\eta_2)_t \end{array} \right. \quad z=-h$$

3. Quartic Dispersion Relation

A doubly-infinite set of surface waves (with horizontal displacements in phase at the layer interface) and internal waves (with horizontal displacements 180º out of phase) [1]. .

The real roots of the dispersion relation ($n=0$) are responsible for the primary surface and internal gravity waves, while the imaginary separation variables for $n \geq 1$ describe both progressive and spatially decaying hydro-acoustic modes.

$$\lambda_n^2\left(R+\alpha_n T_n \hat{T}_n\right) - \lambda_n R\left(T_n + \alpha_n \hat{T}_n\right) + (R-1)\alpha_n T_n \hat{T}_n = 0$$

$$\beta_{w,n}^2 = k_n^2 - \frac{\omega^2}{c^2},\; \beta_{s,n}^2 = k_n^2 - \frac{\omega^2}{c_s^2},\; \alpha_n = \frac{\beta_{s,n}}{\beta_{w,n}},\; \lambda_n = \frac{\omega^2}{g\beta_{w,n}}$$

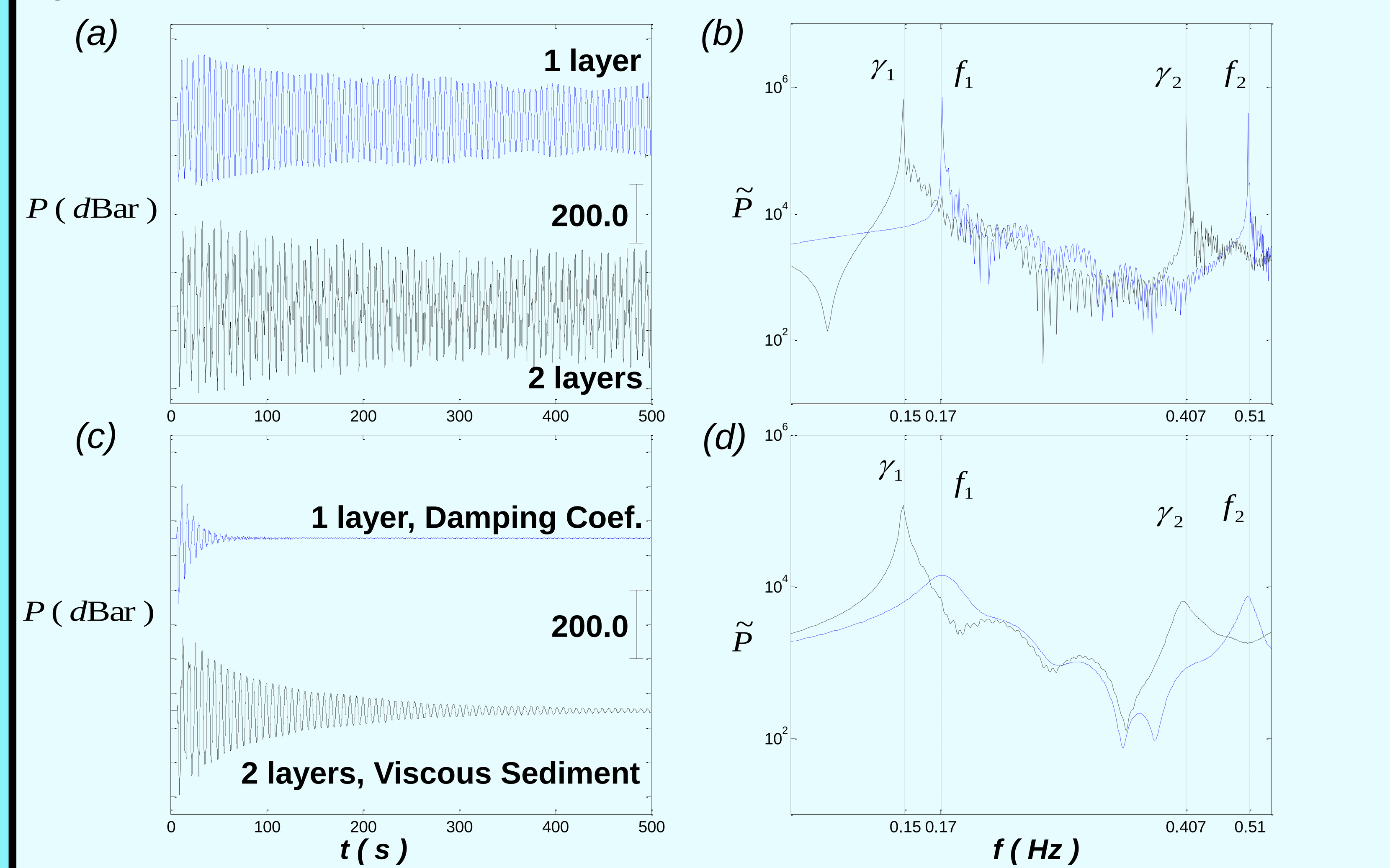
$$T_n = \tanh(\beta_{w,n} h),\; \hat{T}_n = \tanh(\beta_{s,n} a),\; R = \frac{\rho_s}{\rho}$$

4. Role of Underlying Sedimentary Layer

$$f_{(n)} = (2n-1)\frac{c}{4h}; \quad n = 1,2,3,...$$ Cut off frequency for 1 Layer [2,3]

$$\tan\left[\frac{(2\pi\,\gamma_{(n)} h)}{c}\right]\tan\left[\frac{(2\pi\,\gamma_{(n)} a)}{c_s}\right] = \frac{\rho_s\, c_s}{\rho\, c}$$ Cut off frequency for 2 Layers

Sample Computation: Distance from epicenter *x = 96 km*, Constant depth, *h=2200 m, a=1000 m, c= 1500 m/s,* c_s *= 2000 m/s, ρ=1000 kg/ m*^{3}, ρ_s = 1850 kg/m^3, unit source semi-length *b=112 km* and rising time τ *=1 s.*



(a,b) One layer compressible water model (blue) and a coupled model of compressible water & inviscid compressible sediment (black), *μ = 0*.
(c,d) One layer compressible water model with partial reflection bottom boundary condition (blue) and a coupled model of compressible water & viscous compressible sediment, *μ = 2 x 10*^{8} *Pa s* (black).

5. Mild Slope Equation for Dispersive Weakly Compressible fluids (MSEDWC)

The upper and lower layers potential are expanded using separation of variables technique:

$$\begin{cases} \Phi(x,y,z,t) = \sum_{n=0}^{\infty}\Phi_n(x,y,z,t) = \sum_{n=0}^{\infty}\psi_n(x,y,t)\,M_n(z) \\ Q(x,y,z,t) = \sum_{n=0}^{\infty}Q_n(x,y,z,t) = \sum_{n=0}^{\infty}\psi_n(x,y,t)\,N_n(z) \end{cases}$$

Integration over the depth yields:

$$I_{mn} = \int_{-h}^{0} M_m(z)\,M_n(z)\,dz; \quad K_{mn} = \int_{-h_s}^{-h} N_m(z)\,N_n(z)\,dz$$

The eigenfunctions M_n and N_n form a complete Sturm-Liouville basis subject to the orthogonality constraint $I_{mn}+ R\,K_{mn}= 0$ for $m \neq n$.

Using Leibniz' rule and appropriate boundary conditions, neglecting second-order terms in interfacial and substrate slope and staying within the classic mild-slope framework, the expressions for the two layers are derived [4].

Combining Upper and Lower expression taking the advantage of orthogonality within the spatial derivative terms, and making use the interfacial kinematic and dynamic boundary conditions, we obtain the desired mild slope equation [1]:

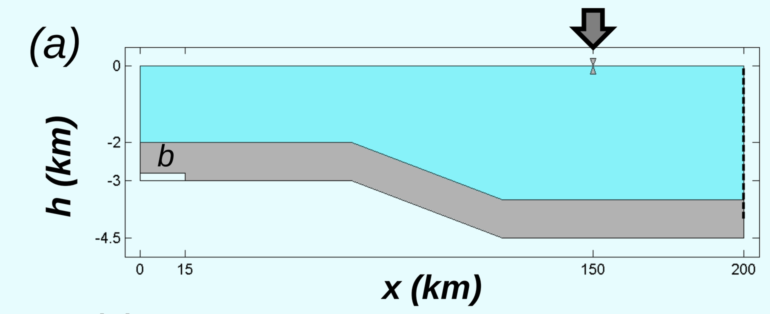
$$\left(I_2^m\,\psi_{m,t}\right)_t - \nabla_h . \left(I_1^m\,\nabla_h \psi_m\right) + \left(\omega^2\, I_2^m - k_m^2\, I_1^m\right)\psi_m + 2\,R\,\varepsilon\,\frac{\omega}{c_s^2}\,K_n\,\psi_{m,t} = D_1^m\, h_t + D_2^m\, h_{s,t}$$

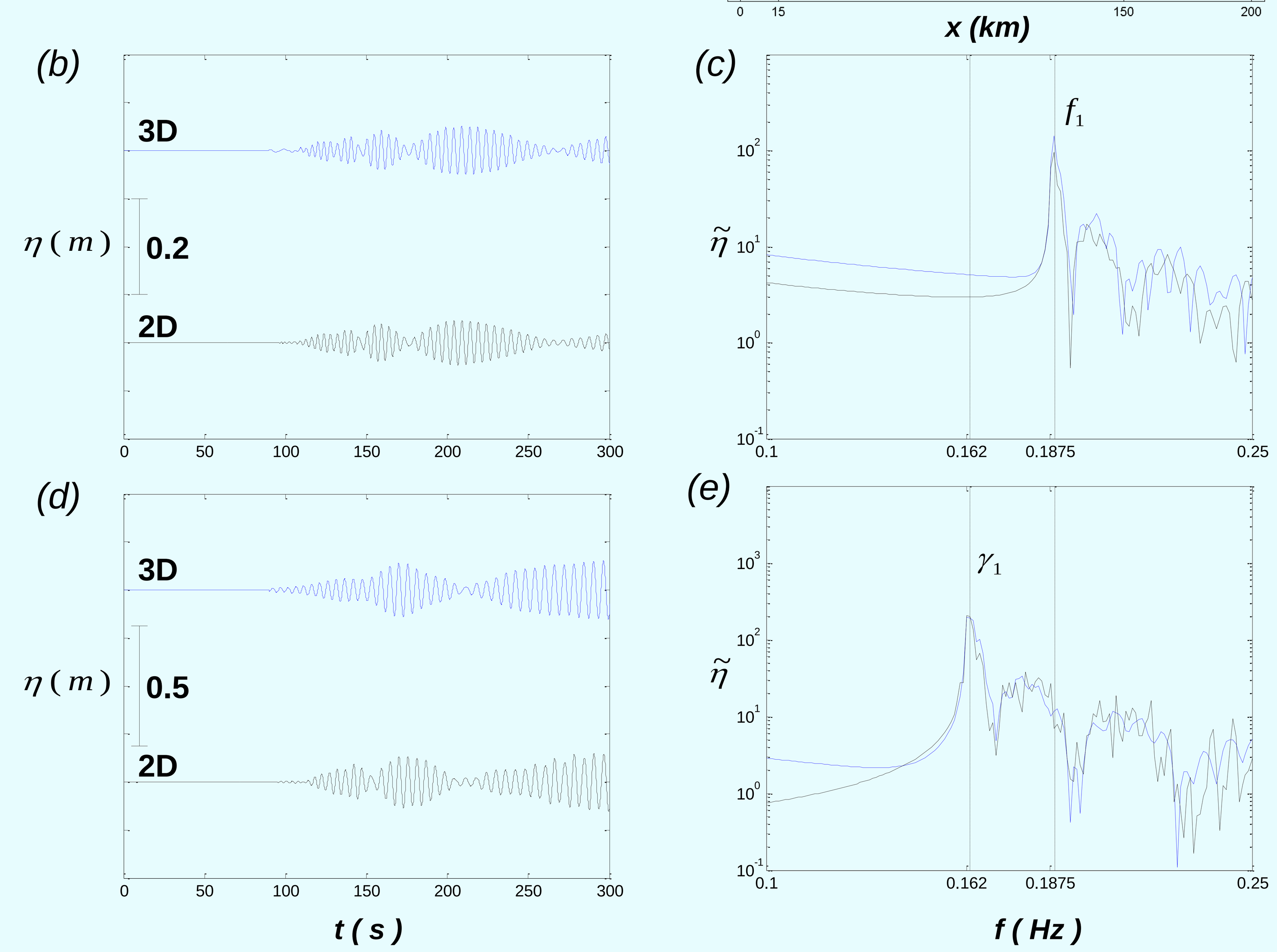
MSEDWC

$$I_1^m = I_{mm} + R\,K_{mm}; \quad I_2^m = \frac{I_{mm}}{c^2} + R\frac{K_{mm}}{c_s^2} + \frac{1}{g}; \quad D_1^m = -\left[M_m - R\,N_m\right]_{(-h)}; \quad D_1^m = -\left[R\,N_m\right]_{(-h_s)}$$

6. Depth Integrated Model Validation

Sample Computation:
Varying sea bottom and sediment thickness. ***(a)*** The computational domain.





Results for FSE time series at *150 km* from tsunamigenic source from 3D (blue) and 2D models (black). ***(b,c)*** Results for impermeable sea bottom and ***(d,e)*** for coupled model.

7. Conclusions

❖Assumption of compressible sedimentary layer can explain the mismatch between spectrum peaks, $f_{(n)}$ and $\gamma_{(n)}$ calculated from rigid bottom and coupled models.

❖Interaction between water column and porous medium leads to lowering of spectral peaks and wave attenuation.

❖We derived a depth integrated equation valid for varying water depth and sediment thickness to overcome the computational difficulties of three-dimensional models and limits of available theoretical solutions applicable for complex geometries.

❖The correct detection of hydro-acoustics waves in a real ocean consisting of a variable-depth water column overlying a sediment layer could enhance significantly the efficiency and promptness of Tsunamis Early Warning Systems (TEWS).

8. Acknowledgements/References

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