Landslide Generated Waves
- Smoothed Particle Hydrodynamics (SPH) Model for Soil–Water Coupling

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Benchmark Problem #5
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abstract
We simulate the generation of a landslide-induced impulse wave with a newly-developed soil–water coupling model in the smoothed particle hydrodynamics (SPH) framework. The model includes an elasto–plastic constitutive model for soil, a Navier–Stokes equation based model for water, and a bilateral coupling model at the interface. The model is tested with simulated waves induced by a slow and a fast landslide. Good agreement is obtained between simulation results and experimental data. The generated wave and the deformation of the landslide body can both be resolved satisfactorily. All parameters in our model have their physical meaning in soil mechanics and can be obtained from conventional soil mechanics experiments directly. The influence of the dilatancy angle of soil shows that the non-associated flow rule must be selected, and the value of the dilatancy angle should not be chosen arbitrarily, if it is not determined with relative experiments.
1. Introduction

• Experimental setup

Viroulet et al (2014)

• Test Cases:

Case 1: $D = 1.5$ mm, $H = 14.8$ cm, $L = 11$ cm
Case 2: $D = 10$ mm, $H = 15$ cm, $L = 13.5$ cm

<table>
<thead>
<tr>
<th>granular material</th>
<th>diameter (mm)</th>
<th>$\theta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>small glass beads</td>
<td>1.5</td>
<td>$25.7^\circ \pm 0.9^\circ$</td>
</tr>
<tr>
<td>medium glass beads</td>
<td>4</td>
<td>$23.3^\circ \pm 0.8^\circ$</td>
</tr>
<tr>
<td>large glass beads</td>
<td>10</td>
<td>$20.1^\circ \pm 1.2^\circ$</td>
</tr>
<tr>
<td>aquarium sand</td>
<td>4</td>
<td>$37.3^\circ \pm 0.6^\circ$</td>
</tr>
</tbody>
</table>

Table 1: Mean particle diameter, $d$, and critical angle of avalanche $\theta_c$ for the four different granular media.
2. Numerical Model

• Model for Water

➢ Governing equations:

\[
\frac{d \rho}{dt} = -\rho \frac{\partial v_\beta}{\partial x_\beta}
\]

\[
\frac{dv_\alpha}{dt} = \frac{1}{\rho} \frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} + g
\]

\[
\sigma_{\alpha\beta} = -p \delta_{\alpha\beta} + \tau_{\alpha\beta}
\]

\[
P = B[(\frac{\rho}{\rho_0})^\gamma - 1]
\]

➢ SPH form:

\[
\frac{D \rho_i}{Dt} = \sum_{j=1}^{N} m_j (v_i - v_j) \cdot \nabla_i W_{ij}
\]

\[
\frac{D v_\alpha_i}{Dt} = \sum_{j=1}^{N} m_j \left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_j \right) \cdot \nabla_i W_{ij} + g^\alpha
\]

Artificial viscosity term

\[
\Pi_j = \begin{cases} 
-\alpha c_i \frac{\mu_j}{\rho_j}, & v_j \cdot r_j < 0 \\
0, & v_j \cdot r_j \geq 0
\end{cases}
\]

Weight function or kernel

\[
W(r,h) = \alpha_0 \begin{cases} 
1 - \frac{3}{2}q^2 + \frac{3}{4}q^4, & 0 \leq q \leq 1 \\
\frac{1}{4}(2-q)^3, & 1 \leq q \leq 2 \\
0, & q \geq 2
\end{cases}
\]
2. Numerical Model


\[ \dot{\varepsilon}^{\alpha\beta} = \frac{1}{2} \left( \frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha} \right) \]

**Total strain rate tensor**

\[ \dot{\varepsilon} = \frac{\dot{S}^{\alpha\beta}}{2G} + \frac{1-2\nu}{3E} \dot{\sigma}^{\gamma\gamma} \delta^{\alpha\beta} \]

**Elastic strain rate tensor**

**Generalized Hooke’s law**

\[ \dot{\varepsilon}_p = \lambda \frac{\partial g}{\partial \sigma^{\alpha\beta}} \]

**Plastic strain rate tensor**

**Plastic flow rule**

\[ \dot{\varepsilon}_{\alpha\beta} = \frac{\dot{S}^{\alpha\beta}}{2G} + \frac{1-2\nu}{3E} \sigma^{\gamma\gamma} \delta^{\alpha\beta} + \lambda \frac{\partial g}{\partial \sigma^{\alpha\beta}} \]

**Total strain rate tensor**
2. Numerical Model

- Model for Soil

**Drucker–Prager yield criterion**

**Yield condition**

\[ f(I_1, J_2) = \sqrt{J_2} + \alpha_\phi I_1 - k_c = 0 \]

**Plastic potential function**

\[ g(I_1, J_2) = \sqrt{J_2} + \alpha_\psi I_1 - \text{constant} \]

**Constitutive equations**

\[ \frac{D\sigma_{i}^{\alpha\beta}}{Dt} = \sigma_i^{\alpha\gamma} \delta_{i}^{\gamma\gamma} + \sigma_i^{\gamma\beta} \delta_{i}^{\gamma\alpha} + 2Ge_i^{\alpha\beta} + Ke_i^{\gamma\gamma} \delta_i^{\alpha\beta} - \dot{\lambda}_i \left[ 3\alpha_\psi K \delta^{\alpha\beta} + \frac{G}{\sqrt{J_2}} s_i^{\alpha\beta} \right] \]

**Plastic multiplier**

\[ \dot{\lambda}_i = \begin{cases} 3\alpha_\psi K \dot{e}_i^{\gamma\gamma} + (G/\sqrt{J_2}) s_i^{\alpha\beta} \dot{e}_i^{\alpha\beta} & f(I_1, J_2) = 0 \\ 9\alpha_\phi \alpha_\psi K + G & f(I_1, J_2) < 0 \\ 0 & \text{otherwise} \end{cases} \]
2. Numerical Model

- Model for Soil

Constitutive equations in SPH form:

\[
\frac{D\sigma_{i}^{\alpha \beta}}{Dt} = \sigma_{i}^{\alpha \beta} \dot{\omega}^{\alpha \beta} + \sigma_{i}^{\alpha \beta} \dot{\omega}_{i}^{\alpha \beta} + 2G\dot{e}_{i}^{\alpha \beta} + K\varepsilon_{i}^{\alpha \beta} \dot{\epsilon}_{i}^{\alpha \beta} - \dot{\lambda}_{i} \left[ 3\sigma_{i}^{\lambda \kappa} \dot{\epsilon}_{i}^{\lambda \kappa} + \frac{G}{\sqrt{J_{2}}} s_{i}^{\alpha \beta} \right]
\]

\[
\dot{\epsilon}_{i}^{\alpha \beta} = \frac{1}{2} \left[ \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} (v_{j}^{\alpha} - v_{i}^{\alpha}) \frac{\partial W_{ij}}{\partial x_{j}^{\beta}} + \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} (v_{j}^{\beta} - v_{i}^{\beta}) \frac{\partial W_{ij}}{\partial x_{j}^{\alpha}} \right]
\]

\[
\dot{\epsilon}_{i}^{\alpha \beta} = \frac{1}{2} \left[ \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} (v_{j}^{\alpha} - v_{i}^{\alpha}) \frac{\partial W_{ij}}{\partial x_{j}^{\beta}} - \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} (v_{j}^{\beta} - v_{i}^{\beta}) \frac{\partial W_{ij}}{\partial x_{j}^{\alpha}} \right]
\]

\[
K = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}
\]

Governing equations in SPH form:

\[
\frac{D\rho_{i}}{Dt} = \sum_{j=1}^{N} m_{j} (v_{j}^{\alpha} - v_{i}^{\alpha}) \frac{\partial W_{ij}}{\partial x_{i}^{\alpha}}
\]

\[
\frac{Dv_{i}^{\alpha}}{Dt} = \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} \left( \frac{\sigma_{i}^{\alpha \beta}}{\rho_{i}} + \sigma_{j}^{\alpha \beta} - \Pi_{i}^{\alpha \beta} + F_{i}^{\alpha \beta} R_{i}^{\alpha \beta} \right) \frac{\partial W_{ij}}{\partial x_{i}^{\beta}} + g^{\alpha}
\]

For 2D simulations:

\[
\alpha_{\varphi} = \frac{\tan \varphi}{\sqrt{9 + 12 \tan^{2} \varphi}}
\]

\[
k_{c} = \frac{3c}{\sqrt{9 + 12 \tan^{2} \varphi}}
\]

\[
\alpha_{\psi} = \frac{\tan \psi}{\sqrt{9 + 12 \tan^{2} \psi}}
\]

Artificial stress tensor:

\[
R^{\alpha \beta} = \begin{cases} 
-\frac{\sigma^{\alpha \beta}}{\rho} & \sigma^{\alpha \beta} > 0 \\
0 & \sigma^{\alpha \beta} \leq 0 
\end{cases}
\]
3. Test cases and results

- Numerical setup

> Two treatments for landslide density

\[ \rho_s = (1-n) \rho_g + n \rho_w = 1900 \text{kg m}^{-3} \]

\[ \rho_s = (1-n) \rho_g = 1500 \text{kg m}^{-3} \]

- Values of Soil Parameters for Simulations

<table>
<thead>
<tr>
<th>Cases</th>
<th>( \rho_g ) (kg m(^{-3}))</th>
<th>( n ) (%)</th>
<th>( c ) (kPa)</th>
<th>( \varphi ) (°)</th>
<th>( \psi ) (°)</th>
<th>( E ) (MPa)</th>
<th>( U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>2500</td>
<td>40</td>
<td>0</td>
<td>25.7</td>
<td>0</td>
<td>20</td>
<td>0.3</td>
</tr>
<tr>
<td>Case2</td>
<td>2500</td>
<td>40</td>
<td>0</td>
<td>20.1</td>
<td>0</td>
<td>20</td>
<td>0.3</td>
</tr>
<tr>
<td>Case3</td>
<td>2500</td>
<td>40</td>
<td>0</td>
<td>25.7</td>
<td>0</td>
<td>20</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- Particles’ amount for Simulations

<table>
<thead>
<tr>
<th>Cases</th>
<th>( dp ) (m)</th>
<th>Soil</th>
<th>Water</th>
<th>Bound</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>0.002</td>
<td>1600</td>
<td>42561</td>
<td>9706</td>
<td>53867</td>
</tr>
<tr>
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<td>2401</td>
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<td>Case3</td>
<td>0.002</td>
<td>2695</td>
<td>40227</td>
<td>9690</td>
<td>52612</td>
</tr>
</tbody>
</table>
3. Test cases and results

- Case 3

  - Left: snapshots of the experiment with $m_s = 3$kg, reprinted from Viroulet et al (2013)

  - Right: simulation results,
  It should be noticed that the figures at right side have larger zone than the left ones.

- Comparison
  flow field
  soil configuration
3. Test cases and results

- Case 1

\[ \rho_s = 1900 \text{kg m}^{-3} \]

\[ \rho_s = 1500 \text{kg m}^{-3} \]

- Simulation-Rho=1500
- Simulation-Rho=1900
- Experiment

\[ t = 0.20s \]

\[ t = 0.40s \]

\[ t = 0.60s \]
3. Test cases and results

- Case 2

\( \rho_s = 1900 \text{ kg m}^{-3} \)

\( \rho_s = 1500 \text{ kg m}^{-3} \)

\( t = 0.20s \)

\( t = 0.40s \)

\( t = 0.60s \)
4. Discussion

- Convergence tests

- Numbers of Particles used for simulations

<table>
<thead>
<tr>
<th>Cases</th>
<th>dp (m)</th>
<th>Soil</th>
<th>Water</th>
<th>Bound</th>
<th>Total</th>
<th>Amplitude(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.0015</td>
<td>2809</td>
<td>75578</td>
<td>13062</td>
<td>91449</td>
<td>2.0773</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
<td>1600</td>
<td>42561</td>
<td>9706</td>
<td>53867</td>
<td>1.9850</td>
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<tr>
<td></td>
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<td>729</td>
<td>18935</td>
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</tr>
<tr>
<td></td>
<td>0.004</td>
<td>417</td>
<td>10724</td>
<td>4922</td>
<td>16063</td>
<td>1.5100</td>
</tr>
</tbody>
</table>
4. Discussion

Schematic diagram of fjord-like channel and the deformable landslide

Water surface displacement at different points along the channel

Wave generation process: (a) t=4s (b) t=8s (c) t=12s (d) t=16s (e) t=20s (f) t=24s (g) t=28s (h) t=32s.
4. Discussion

- **DEM method**
  - Case 2

DEM-SPH model by Canelas et al (2016)

Case 2:
- D=1cm
- L=13.5cm

Landslide → Continuum body

![Simulation graphs and Continuum body images](attachment:image.png)