Introduction

In the present study, two tsunamigenic landslide models are examined by a number of benchmark problems. The simulated cases are several sets of experimental data which consider landslide as either a solid or a deformable sliding mass. The first model, LS3D model, is applied to model the tsunami waves caused by a solid landslide and the second one, 2LCMFlow model, is benchmarked with deformable sliding masses. The following sections include a brief description of these benchmarked models.

LS3D model:

The LS3D model simulates landslide tsunamis by solving the following fourth-order Boussinesq-type equations:

$$
\begin{aligned}
&\frac{1}{\varepsilon}h_t + \zeta_t + \nabla.\Big\{ (\varepsilon\zeta + h)\mathbf{u_0} \\
&\quad + \mu^2\left[\tfrac{-1}{6}(\varepsilon^3\zeta^3 + h^3)\mathbf{A} + \tfrac{1}{2}\tilde{z}^2(\varepsilon\zeta + h)\mathbf{A} - \tfrac{1}{2}(\varepsilon^2\zeta^2 - h^2)(\nabla.B) + \tilde{z}^2(\varepsilon\zeta + h)(\nabla.B)\right] \\
&\quad + \mu^4\Big[\tfrac{1}{120}(\varepsilon^5\zeta^5 + h^5)\nabla(\nabla.\mathbf{A}) - \tfrac{1}{24}(\varepsilon\zeta + h)\tilde{z}^4\nabla(\nabla.\mathbf{A}) - \tfrac{1}{12}(\varepsilon^3\zeta^3 + h^3)\nabla(\nabla.(\tilde{z}^2.\mathbf{A})) \\
&\qquad + \tfrac{1}{4}(\varepsilon\zeta + h)\tilde{z}^2\nabla(\nabla.(\tilde{z}^2\mathbf{A})) + \tfrac{1}{24}(\varepsilon^4\zeta^4 - h^4)\nabla(\nabla.(\nabla B)) - \tfrac{1}{6}(\varepsilon\zeta + h)\tilde{z}^3\nabla(\nabla.(\nabla B)) \\
&\qquad - \tfrac{1}{6}(\varepsilon^3\zeta^3 + h^3)\nabla(\nabla.(\tilde{z}\nabla B)) + \tfrac{1}{2}(\varepsilon\zeta + h)\tilde{z}\nabla(\nabla.(\tilde{z}\nabla B)) \\
&\qquad + \tfrac{1}{2}(\varepsilon^2\zeta^2 - h^2)\nabla C - (\varepsilon\zeta + h)\tilde{z}\nabla C\Big] \Big\} = O(\varepsilon^6, \mu^6)
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&\mathbf{u_{0t}} + \varepsilon(\nabla.\mathbf{u_0})\mathbf{u_0} + \varepsilon\left(w_1|_{z=0}\right)\mathbf{u_{0z}} \\
&\quad + \mu^2\left[\mathbf{u_{1t}}|_{z=0} + \varepsilon(\nabla.\left(\mathbf{u_1}|_{z=0}\right))\mathbf{u_0} + \varepsilon(\nabla.\mathbf{u_0})\left(\mathbf{u_1}|_{z=0}\right) + \varepsilon\left(w_2|_{z=0}\right)\mathbf{u_{0z}} + \left(w_1|_{z=0}\right)\left(\mathbf{u_{1z}}|_{z=0}\right)\right] \\
&\quad + \mu^4\Big[\mathbf{u_{2t}}|_{z=0} + \varepsilon(\nabla\left(\mathbf{u_2}|_{z=0}\right))\mathbf{u_0} + \varepsilon(\nabla.\left(\mathbf{u_1}|_{z=0}\right))\left(\mathbf{u_1}|_{z=0}\right) + \varepsilon(\nabla.\mathbf{u_0})\left(\mathbf{u_2}|_{z=0}\right) \\
&\quad + \varepsilon\left(w_2|_{z=0}\right)\left(\mathbf{u_{1z}}|_{z=0}\right) + \left(w_1|_{z=0}\right)\left(\mathbf{u_{2z}}|_{z=0}\right)\Big] \\
&\quad + \nabla\left(P|_{z=0}\right) = O(\varepsilon^6, \mu^6)
\end{aligned}
\tag{2}
$$

Eq. 1 and 2 represent the continuity and the depth-averaged momentum equations in the two horizontal x and y directions, respectively. $\varepsilon = \frac{a_0}{h_0}$ and $\mu = \frac{h_0}{L_0}$ are two indexes indicating wave nonlinearity and dispersive behaviour. a_0, L_0, and h_0 stand for a characteristic wave amplitude, wave length and water depth, respectively. The subscripts represent the partial derivative (e.g. $h_t = \frac{\partial h}{\partial t}$). t is time, h water depth, ζ the water surface fluctuations, p the water pressure, and $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ the horizontal gradient vector. The velocity domain components $\mathbf{u} = (u, v)$ and w which respectively represent the vector of the horizontal velocity components and the vertical velocity component in the z direction, are expanded into

$$\mathbf{u} = \mathbf{u_0} + \mu^2.\mathbf{u_1} + \mu^4.\mathbf{u_2} \tag{3}$$

$$w = \mu^2.w_1 + \mu^4.w_2 \tag{4}$$

in perturbation analysis with μ^2 as the basic small parameter. $\tilde{z}$ is a characteristic variable depth defined as a weighted average of two distinct water depths z_a and z_b based on $\tilde{z} = [\beta.z_a + (1-\beta).z_b]$. β is an optimized weighting parameter. Moreover, $\mathbf{A} = \nabla(\nabla.\mathbf{u}_0)$, $\mathrm{B} = \nabla.(h\mathbf{u}_0) + \frac{1}{\varepsilon}h_t$, and $\mathrm{C} = f(\mathbf{A}, \mathrm{B})$. A schematic definition of the model parameters can be observed in Fig. 1.

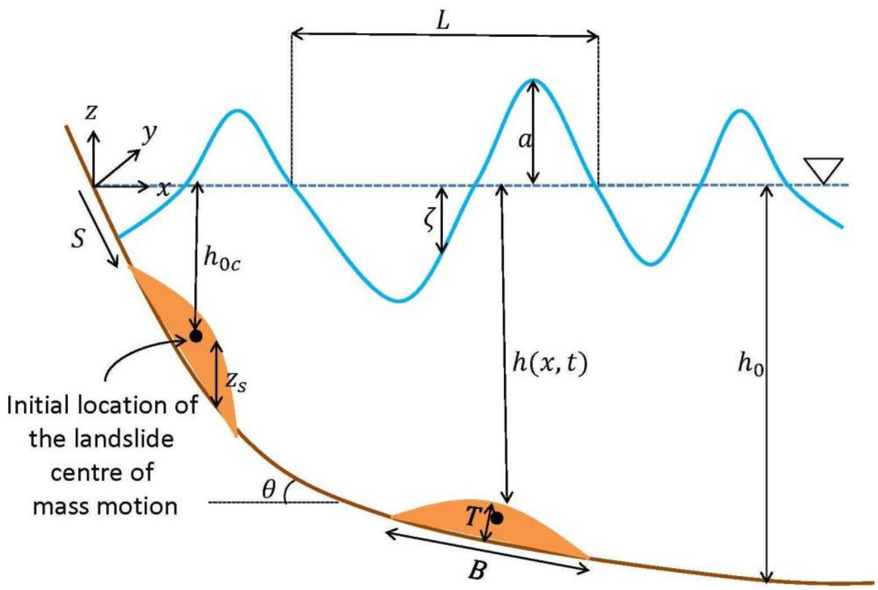

Fig. 1 The LS3D model parameters and assumptions

A sixth-order multi-step finite difference method was utilized for spatial discretization and a sixth-order Runge–Kutta method was implemented for temporal discretization of the higher-order depth-integrated governing equations and boundary conditions.

The LS3D model describes landslide as a time variable bottom boundary with a rigid hyperbolic-shape geometry. The law of the mass motion is

$$S(t) = S_0 \ln(\cosh\frac{t}{t_0}) \tag{5}$$

Where S is the location of landslide centre of mass motion parallel to the slope, $S_0 = u_t^2/a_0$, and $t_0 = u_t/a_0$. u_t is the terminal velocity of the sliding mass and a_0 is its initial acceleration defined as

$$u_t = \sqrt{gB}.\sqrt{\frac{\pi(\gamma-1)}{2C_d}.\sin\theta}\ , \quad a_0 = g\frac{\gamma-1}{\gamma+C_m}\sin\theta \tag{6}$$

where $\gamma=\rho_s/\rho_w$, B is the length of the sliding mass along the inclined bed, C_d is the drag coefficient, C_m is the added mass coefficient, θ is the bed slope and g is the gravitational acceleration. ρ_s and ρ_w are the landslide and the water densities, respectively. The mass motion equation is obtained by transforming Eq. 5 from the bottom direction to the Cartesian coordinate (x, y, z) direction. Accordingly, the time variable bottom boundary is obtained as

$$h(x,t) = h_0(x) - 0.5T\left(1 + tanh\left(\frac{x-x_l(t)}{S}\right)\right)\left(1 - tanh\left(\frac{x-x_r(t)}{S}\right)\right) \quad (7)$$

where $x_l(t) = x_c(t) - 0.5Tcos\theta$ and $x_r(t) = x_c(t) + 0.5Tcos\theta$ are the locations of the rear and the front ends of the sliding mass, respectively. $x_c(t)$ is the location of the sliding mass centre, T is the maximum mass thickness and S=o.5/$cos\theta$. Eq. 7 estimates the location of the sliding mass center at each time step.

For the three-dimensional conditions, a truncated hyperbolic secant function of x and y with a specific truncation ratio, r, as introduced by Enet et al. (2003), is applied to describe the landslide model geometry.

$$\begin{cases} z_s(x,y) = \frac{T}{r}(sech(K_w x)\,.\,sech(K_B y) - (1-r)) & z_s \geq 0 \\ K_w = \frac{2}{w}\text{asech}\left(\frac{1-r}{r}\right), \quad K_B = \frac{2}{B}\text{asech}\left(\frac{1-r}{r}\right) \end{cases} \quad (8)$$

where z_s is the thickness of the sliding mass moving along the bed. The specific truncation ratio can be modified according to the real geometry of the sliding mass. The effects of the solid block movements on the water surface fluctuations is inserted into the model equations through the kinematic boundary condition of the bed which is

$$h_t + \mathbf{u}.\nabla h = -w \quad z = -h \quad (9)$$

The original LS3D was able to model submarine landslides (Ataie-Ashtiani and Najafi-Jilani 2008). In 2011, the model was extended to handle subaerial landslide cases by Ataie-Ashtiani and Yavari-ramshe (2011) based on the method of Lynett and Liu (2005). According to Eqs. 4 and 5, the kinematic characteristics of the sliding mass depend on u_t, and a_s. For subaerial cases, landslide velocity must be altered to include the aerial acceleration. Accordingly, they formulated the sliding velocity as a weighted average of the aerial and submerged velocities, where the weighting parameter is based on the fraction of the submerged volume. Thus, the slope-parallel velocity of the slide is given by

$$f_s u_s + f_a g t sin\theta \quad (10)$$

The coefficients f_s and f_a represent the submerged and the aerial volume fractions of the landslide, respectively. The time-dependent velocity of a submerged landslide, u_s, is calculated as in (Grilli et al. 2002)

$$u_s = u_t \tanh(\frac{t}{t_0}) \tag{11}$$

This linear combination of the aerial and submerged velocities is used instead of terminal velocity, u_t, in Eq. 6.

The model inputs contain the basin topography, the still water surface level, h_0, reflection factor, F (representing the reflection percentage of the lake borders), the geometrical properties of the landslide including the sliding mass length, B, width, w, and maximum thickness, T, the relative density γ, the slide initial depth, h_{0c}, the sliding slope angle, θ, the drag coefficient, C_d, and the added mass coefficient, C_m. The LS3D model has been successfully validated using two sets of experimental data on submarine (Ataie-Ashtiani and Najafi-Jilani 2008) and subaerial (Ataie-Ashtiani and Nik-Khah 2008) landslides. It has also been applied to estimate the landslide tsunamis, wave runup, and dam overtopping for two real cases of Shafarood (Ataie-Ashtiani and Najafi-Jilani 2007; Ataie-Ashtiani and Yavari-Ramshe 2011) and Maku (Yavari-Ramshe and Ataie-Ashtiani 2009; Ataie-Ashtiani and Yavari-Ramshe 2011) dam reservoirs.

2LCMFlow model:

The 2LCMFlow model developed by Yavari-Ramshe and Ataie-Ashtiani (2015) solves the shallow water equations (SWEs), incompressible Euler equations, for a two-layer flow including a layer of granular material moving beneath a layer of water based on a state of the art Roe-type finite volume method introduced by Yavari-Ramshe et al. (2015). The sliding mass is described as a Coulomb mixture; a two-phase mixture of water and solid grains where its interaction with the bottom follows a Coulomb-type friction law and the normal and longitudinal stresses of the solid phase are related with the earth pressure coefficient, K. The final system of mathematical equations of this two-layer Coulomb mixture flow (2LCMFlow) model is

$$\begin{cases} h_{1t} + (q_1 cos\theta)_x = 0 \\ q_{1t} + \left(h_1 u_1^2 cos\theta + g\frac{h_1^2}{2}cos^3\theta\right)_x = -gh_1 cos\theta b_x + g\theta_x \frac{h_1^2}{2} sin\theta cos^2\theta \\ \qquad -gh_1 cos\theta (h_2 cos^2\theta)_x \\ h_{2t} + (q_2 cos\theta)_x = 0 \\ q_{2t} + \left(h_2 u_2^2 cos\theta + g\Lambda_2 \frac{h_2^2}{2} cos^3\theta\right)_x = -gh_2 cos\theta b_x + g\theta_x \frac{h_2^2}{2} sin\theta cos^2\theta \\ \qquad -rgh_2\Lambda_1 cos\theta(h_1 cos^2\theta)_x + \frac{\Im}{cos\theta} \end{cases} \tag{12}$$

where subscript 1 and 2 represent the water and the granular layers, respectively. θ is the local bed slope and $b(x)$ is the bottom topography. Moreover, $q = hu$, , $\Lambda_1 = \lambda_1 + K(1 - \lambda_1)$, and $\Lambda_2 = r\lambda_2 + K(1 - r\lambda_2)$ where $K = 2\left(1 - sgn(u_{2x})\sqrt{1 - \left(\frac{cos\phi}{cos\delta_{mod}}\right)^2}\right)/cos^2\phi - 1$ and $tan\delta_{mod} = tan\delta - \lambda' K h_{2x}$. $r = \frac{\rho_2}{\rho_1}$ is the relative density of the landslide. The parameters ϕ and δ stand for the internal and the basal friction angles of the granular layer, respectively. δ_{mod} is the dynamically modified basal friction angle. ρ_1 and ρ_2 are the water and the landslide densities, respectively, and λ' is a constant. Finally, $\Im$ stands for the Coulomb friction term defined as

$$\begin{cases} \Im = -(g(1-r)h_2 cos^2\theta + h_2 u_2^2 cos\theta (sin\theta)_x)\frac{q_2}{|q_2|} tan\delta_{mod} & |\Im| \geq \sigma_c \\ q_2 = 0 & |\Im| < \sigma_c \end{cases} \tag{13}$$

where σ_c is a basal critical stress which is defined based on δ_0, the angle of repose of the granular material, as $g(1-r)h_2 cos\theta tan\delta_0$. Eq. 13 stops the landslide from moving when its angle is less than the angle of repose. The constitutive structure of the sliding material is defined using two coefficients λ_1 and λ_2 which distribute the water layer pressure between the solid and the fluid phases of the second layer on the interface and along the second layer, respectively (Yavari-Ramshe and Ataie-Ashtiani 2015), as

$$\begin{cases} P^f_{2zz} = \lambda_1 \rho_1 h_1 cos\theta + \lambda_2 \rho_1 (h_2 - z) cos\theta \\ P^s_{2zz} = (1-\lambda_1)\rho_1 h_1 cos\theta + (\rho_2 - \lambda_2 \rho_1)(h_2 - z) cos\theta \end{cases} \tag{14}$$

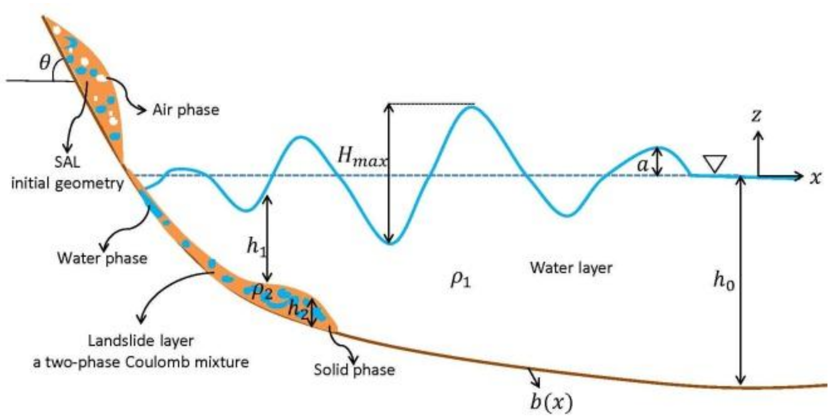

Fig. 2 Schematic definition of the 2LCMFlow model parameters and assumptions

In Eq. 14, P_{zz} is the normal stress and the superscripts f and s stands for the fluid and the solid phases of the second layer (the sliding mass), respectively. The 2LCMFlow is able to capture the simultaneous appearance of the static/flowing regions along the landslide motion. The model is also capable of simulating the interactions between water and a variety of granular material with different water content from rockslide and dry cohesion-less material to loose and muddy flows based on the considered rheological and constitutive structures. A schematic of the model parameters is illustrated in Fig. 2. The 2LCMFlow model is successfully validated against the same experiments of Ataie-Ashtiani and Najafi-Jilani (2008) and Ataie-Ashtiani and Nik-Khah (2008).