DEPARTMENT OF POLITICAL SCIENCE
AND
INTERNATIONAL RELATIONS
Research Methods Posc 302

THE CHI SQUARE TEST
(Examples and Applications)

I. TODAY'S SESSION:
- A. The chi square test of statistical independence
- B. Research designs for final projects
- C. Writing tips:
  - 1. Look carefully at these sentences:
    - i. "In Philadelphia's 1999 election for mayor between Katz and Street, questioned whether the race of supporters would have an effect on the turnout of the election. The chart below focuses on a similar idea."
    - ii. A for effort and originality.
    - iii. But read the two sentences out loud. Do they "parse" or make sense? Why the comma between "Street" and "questioned"? Is "...on the turnout of the election" awkward?
    - iv. It's a **table,** not a **chart**.
    - v. An alternative: "The 1999 Philadelphia election for mayor raised a question about raise and turnout: do whites and blacks participate at the same rate. The table below provides a partial answer."

II. BASIC IDEAS BEHIND THE CHI SQUARE TEST:
- A. Null hypothesis:
  - 1. X and Y are statistically independent. (SEE BELOW.)
- B. Alternative hypothesis:
  - 1. X and Y are statistically related.
- C. "Experimental" procedure"
  - 1. Interview N subjects, record responses to X and Y, and create cross-classification.
  - 2. Observe the departure from statistical independence as measured by **chi square** statistic.
- D. Sampling distribution.
  - 1. The chi square distribution, which we will not explicitly examine, relates outcomes (that is, chi square statistics) to the probability of their occurrence **if there is statistical independence in the population; that is, if H_0 is true**.
- E. Decision rule:
  - 1. For this course we'll reject the null hypothesis if the probability of getting the chi square we observed or a large one is .05 or 1/20 or less.

2. In other words, if our sample result occurs with probability .05 or less, we will reject the null hypothesis and argue that the variables are not statistically independent.

F. Data analysis
1. Obtain an observed cross-tabulation and corresponding chi square statistic.
2. Observe the probability of this chi square.
3. Accept or reject H_0

G. Interpretation
1. What the hypothesis test indicates is the likelihood of statistical independence between in the population.
2. But just because two variables are statistically related we cannot necessarily conclude that there is substantively interesting relationship between them.
3. To do that we need to examine the table carefully to see how and how strongly X and Y are related.

H. Example:
1. The computer program we've been using reports a chi square statistic along with probability.
   i. The interpretation of these numbers is: if there were no relationship between the variables in the population, the probability of observing a chi square of ... or larger would be ...
2. Here's the table we looked at last week (Class 16).

Cells contain: -Column percent -N of cases		attend				
		1 Seldom	2 Moderate	3 Often	4 A lot	*ROW TOTAL*
polviews	1 Liberal	**18.0** 179	**12.3** 69	**12.4** 50	**8.4** 53	*13.6* *351*
	2 Lean L	**14.8** 147	**14.6** 82	**11.7** 47	**8.4** 53	*12.7* *329*
	3 Mod	**40.8** 406	**41.7** 234	**37.6** 151	**37.0** 233	*39.6* *1,024*
	4 Lean C	**14.5** 144	**18.0** 101	**19.4** 78	**18.4** 116	*17.0* *439*
	5 Cons	**12.0** 119	**13.4** 75	**18.9** 76	**27.7** 174	*17.2* *444*
	COL TOTAL	*100.0* *995*	*100.0* *561*	*100.0* *402*	*100.0* *629*	*100.0* *2,587*

Figure 1: Association Between Church Attendance and Ideology

i. The observed chi square for this table is 112.17.

ii. Moreover, the probability reported for the chi square is .0000, which means that **if in the population church attendance was statistically independent of political ideology, the chances of observing a chi square this large or larger would be less than 1 in 10,000.**

iii. Hence we would reject the null hypothesis, conclude that the variables are statistically related, and then examine the percentages to see how (and how much) the variables depart from statistical independence.

III. STATISTICAL INDEPENDENCE*:

A. So far in the semester we've just "verbalized" the meaning of "relationship" and occasionally used figures or examples to get explain the idea.

1. Now let's state the notion of relationship, or more precisely, lack of relationship more formally.

B. Marginal probabilities

1. Consider two categorical variables such as X = number of times a person attends church and Y = self-placement on a liberalism conservatism.

i. Let's denote that probability that a randomly selected person has a particular value on X as P(X = a).

1) Read this statement as "the probability that a person's score on X is a." The letter "a" is just a symbol for some arbitrary value of X.

ii. Similarly, let P(Y = g) be the probability that a randomly drawn person has some value on Y.

1) Again, the letter "g" is just a symbol that means "some value."

iii. In the present example, we might refer to the probability that a person says "never" when asked how often he or she attends church during the month as P(X = "never").

1) Usually, we use the term P(X) to mean "the probability that X equals some value.)

2) And a similar phrase, P(Y) can be used for the probability that Y equals some value.

iv. **These are called marginal probabilities.**

C. Joint probabilities

1. A randomly selected and interviewed individual can be classified on both variables.

i. Example: a person has value X = "never" **and** Y = "very liberal."

ii. Another person might be jointly classified X = "twice a week" and Y = "moderate."

2. We are interested in the probability that a person has a particular value on X and a particular value on Y.
    i. Let's call this probability P(X = aY = g), which can be read "the probability that X = a **and** Y = g), where of course a and g are simply letters that stand for particular values of the variables.
    ii. It's usually easiest and sufficient to just write **P(XY)** to mean the joint probability that X equals a value and Y (simultaneously) equals some value.

D. Statistical independence:
1. Variables X and Y are **statistically independent** if and only if P(XY) = P(X)P(Y).
    i. In words, X and Y are independent if and only if the joint probability that X = a and Y = g equals the product of the marginal probabilities.

E. This is pretty abstract, but fortunately there are zillions of ways to provide an intuitive understanding.
1. Example 1: you toss a coin and simultaneously and independently roll one die with six faces.
    i. The probability of getting "heads" is ½. (Assume the coin is fair.)
    ii. The probability of getting "."(one dot) is 1/6.
    iii. The probability of getting "head" and "." is ½ X 1/6 = 1/12.
    iv. Explanation:
        1) The marginal probability of "X = head" is ½.
        2) The marginal probability of "Y = one" is 1/6.
        3) Since the coin toss and roll are done independently, the joint probability that "X = head" and "Y = one" is the product of the marginal probabilities or ½ X 1/6 = 1/12.
2. Example 2: you draw a card at random from a 52-card deck and independently toss an honest coin.
    i. What is the probability of getting the joint occurrence "king of hearts" **and** "tail."
    ii. Since there is (presumably) only one king of hearts in the deck, the marginal probability of drawing it is 1/52.
    iii. And as we've seen countless times before the probability of tossing a tail in one flip of a fair coin is ½.
    iv. Consequently, the probability P(X = king of hearts, Y = tail) equals P(kind of heart) times P(tail) = 1/52 X ½ = 1/104.
3. Example 3: suppose you interview people from a well studied population of Americans.
    i. In this population there are 55 percent females so the marginal probability of drawing a female is P(gender = female) = P(F) = .55.
    ii. Moreover, the proportion of Republicans in the population is .40

and consequently the probability of interviewing a Republican is P(party = Republican) = P(R) = .40.

iii. Consequently, the joint probability that a randomly selected individual is a female Republican is P(F) X P(R) = .55 X .40 = .22.

F. The chi square statistic lets us test the hypothesis that the values of one variable are independent of the values of another variable.

1. If this situation turns out to be true, we'll say that there is not relationship between them in the population.

IV. SUBSTANTIVE VERSUS STATISTICAL SIGNIFICANCE:

A. When a someone rejects a null hypothesis in favor of an alternative the result is said to be **statistically significant**.

1. Example: if we reject the hypothesis of (statistical) independence between X and Y because the observed chi square occurs with probability of .05 or less, we say that the variables are significantly related.

i. Frequently, the phrase "at the .05 level" is added so the sentence becomes "There is a significant relationship between X and Y at the .05 level."

B. But the term or phrase "significantly related" means only that we have rejected a statistical null hypothesis, namely that X and Y are independent.

C. The relationship may be **substantively** or **theoretically** unimportant or perhaps weak.

1. In other words, rejecting a statistical hypothesis does not mean we have found something of great interest or importance. We may have, but may be not.

D. Here's an example:

1. Suppose we hypothesize that X and Y are statistically independent and collect data to test this proposition.

2. We agree to use the .05 level. That is, we'll calculate a chi square for the observed table based on N observations, and if the probability of getting this result is .05 or less, we'll reject the null hypothesis of independence.

3. Suppose the observed table is the one shown on the next page.

4. Most of us would agree (or we should that there is a very small to non-existent relationship between gender and opinion.

i. The percentage "for" is about the same.

5. Note also that we have only interviewed N = 80 people and that each column contains 40 respondents.

6. Note finally that the chi square is .200 with a reported P of .655. This means that the observed chi square (.200) **is** likely if the variables are statistically independent in the population.

7. So far so good.

Table 1			
Opinion/ Gender	Male	Female	
For	47.5% 190	52.5% 21	
Against	52.5 21	47.5 19	
Totals	100 (**40**)	100 (**40**)	N = **80**
Chi square = .200 (P = .655)			

8. Now suppose we managed to interview 10 times as many people thereby having an N = 800 subjects.

Table 2			
Opinion/ Gender	Male	Female	
For	47.5% 190	52.5% 210	
Against	52.5 210	47.5 190	
Totals	100 (**400**)	100 (**400**)	N = **800**
Chi square = .2.00 (P = .157)			

i. The relationship as measured by the percentages has not changed at all. About 50 percent of both men and women are "for" the policy.

ii. So we would conclude that the substantive relationship remains very weak. Adding respondents hasn't changed the substantive of theoretical interpretation.

iii. But notice that the chi square has increased by a factor of 10.

iv. And the probability of observing this chi square has decreased. It's still above our .05 level or cut off point. Nevertheless, it's seems to suggest that the null hypothesis was close to being rejected.

9. Now suppose we had unlimited riches and could interview without end.
    i. Suppose in fact we interviewed N = 8,000 individuals but that the relationship between gender and opinion did not change.
    ii. The new table might be:

Table 3			
Opinion/ Gender	Male	Female	
For	47.5% 1900	52.5% 2100	
Against	52.5 2100	47.5 1900	
Totals	100 (**4000**)	100 (**4000**)	N = **8000**
Chi square = .20.00 (P = .000)			

    iii. The sample size has been increased once again by 10 so the chi square increases by a factor of 10.
        1) It's increased from 2.0 to 20.0.
        2) **This will always be the case.** As N increases, chi square increases in lockstep, assuming all else is equal.
    iv. So, now we still have weak relationship as measured by the percentages but the chi square is highly (statistically) significant.

E. Summary:
1. In the social sciences we have to separate two ideas:
    i. **Statistical significance**: can a statistical hypothesis be rejected? If so, the result is **statistically significant**.
        1) We use tests of significance like chi square to answer that question.
    ii. Are the results meaningful from a policy or applied or commonsense or theoretical point of view?
        1) We use percentages, magnitudes of differences, and measures of association to answer this question.

V. RESEARCH DESIGNS:

A. Assignment 5 asks you to prepare a brief statement about your choice of variables.
1. You might use this example to help you organize your thinking and response.
2. **But don't outline your design**. Write it out in full.

3. My topic: Social Class Polarization In the United States
   i. I wonder if the lower class differs significantly (in a theoretical sense) from the upper class in terms of economic outlook.
      1) A concern for false consciousness motivates this interest: does the lower class realize that its economic well being has in fact not improved much in the last 20 years and may have even slipped?
4. Data: 1996 American National Election Study
5. Independent variable: I am going to use occupation (re-coded lightly) as the indicator of social class.
   i. The particular item is: v960676 "Pre. Stacked -- R collapsed occupation code"
6. Dependent variables: I am going to investigate class differences on respondents' perceptions of the economy. The main variables are:
   i. v960338 "Pre. Is R much better or worse off financially than a year ago"
   ii. v960340 "Pre. Does R think R will be much better/worse off financially next year."
   iii. v960386 "Pre. R think economy has gotten much better/worse over past year"
   iv. v960388 "Pre. R expects economy to get much better or worse over the next year"
   v. v960389 "Pre. R think the standard of living will be better or worse in 20 years"
   vi. v960391 "Pre. Have the federal government policies made nation's economy much better or worse"
7. Control variables: to make sure that any differences between classes are not due just to education or family income I will control for these two variables.
   i. v960610 "Pre. Summary of R's education"
   ii. v960701 "Pre. R's family income in 1995"
8. If I don't find a relationship between class (as measured by occupation) and some of the dependent variables, I will discard those items. But if any of the dependent variables are related to class, I'll then further investigate by controlling for family income and education. I am especially interested in knowing if the lower class is more or less or equally optimistic about its and the nation's economic future.

VI. TEST YOURSELF:

A. Here are some chi squares and probabilities of observing them under the null hypothesis. For each indicate if you would reject the null hypothesis or not.

1. Chi square = 25.999 P = .049
2. Chi square = 100.89 P = .158
3. Chi square = 3.06 P = .001

B. Here is a cross-tabulation of family income by attitude toward gun control laws.

1. The question was "Would you favor or oppose a law which would require a person to obtain a police permit before he or she could buy a gun?"
2. That table is:

Frequency Distribution						
Cells contain: -Column percent -N of cases		income91				
		1 Poor	2 Low	3 Medium	4 High	*ROW TOTAL*
gunlaw	1 FAVOR	**82.9** 121	**77.1** 138	**83.9** 266	**84.5** 289	***82.7*** *814*
	2 OPPOSE	**17.1** 25	**22.9** 41	**16.1** 51	**15.5** 53	***17.3*** *170*
	COL TOTAL	***100.0*** *146*	***100.0*** *179*	***100.0*** *317*	***100.0*** *342*	***100.0*** *984*
Means		1.17	1.23	1.16	1.15	1.17
Std Devs		.38	.42	.37	.36	.38

Figure 2: Relation Between Gun Control and Income

3. The chi square (and other statistics) are

Summary Statistics					
Eta* =	.07	Gamma =	-.09	Chisq(P) =	5.04 (p= 0.17)
R =	-.04	Tau-b =	-.04	Chisq(LR) =	4.78 (p= 0.19)
Somers' d* =	-.03	Tau-c =	-.04	df =	3

*Column variable treated as the dependent variable.

Figure 3: Chi Square and Other Statistics

4. Use the data to answer these questions:

i. Look at "Chisq (P)." Is there a statistically significant relationship between attitudes toward gun control and family income? Why?

ii. Is there a substantively or practical relationship between these variables?

iii. Ask me if you are not sure.

VII. NEXT TIME:

A. Discussion of research projects

B. More examples of data analysis including multiway tables.

C. Reading:

1. Wattenberg, ***Decline of Political Parties*** (remember this book) provides some substantive background that might be useful in framing the research hypotheses and interpret the results.