A VAR-based factor decomposition to the term structure of treasury bonds

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Abstract: This paper analyses the dimensions of the term structure and explains their associations with macroeconomic parameters that foretell financial crises. Five prominent financial economic indicators were found, collectively explaining the dynamics of bond yields. Data from China's bond market from 2007 to 2019 were used to calibrate our model and hypothesis. The VAR model was adopted to explore in what direction and to what extent the early warning indicators of financial crises may effect changes in the term structure. Given its high complexity and the irregularity of its occurrence in different economies, it is usually a challenging task to quantitatively model financial crises. This challenge has been handled by an indirect approach introduced in the current study, linking the original limited sample size problem to a VAR-based model with abundant time series data. The findings provide an implementable scheme that may be used to design an early warning system of financial crises.

Keywords: term structure of interest rates; financial crisis; bond yield curve; VAR model; variance decomposition.

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1 Introduction

China's economy has undergone rapid growth in the past 40 years. In the meantime, there has emerged a series of problems, including overcapacity in traditional industry (Li et al., 2019), a high number of non-performing loans in the banking sector (Chen et al., 2018), and relatively low efficiency and turnover in the secondary security market (Han et al., 2019), the solution to which is urgently needed for long-term economic growth and financial stability. On the other hand, all economies are increasingly integrated and intertwined, where the economic sentiment in one region will quickly ripple to other parts of the world. Abundant country-specific reviews of a large economic entity suffice to demonstrate the complex and interconnected nature of financial crises across different markets. Since the 1980s, there have been several major financial crises around the world, which have inspired scholars from various fields to endeavour to provide forecasting measures as well as to design monitoring schemes for financial crises. To date, the task is still open, due to different causalities and types of financial crises across different regions. One major challenge from an econometric modelling perspective is that the occurrence of financial crises, despite its significance to various economies, is highly irregular and the total sample size is rather small, limiting conclusions with reasonable statistical significance.

The objective of the current paper is to provide an indirect route towards a theoretically well-founded and operationally easy-to-implement scheme to serve as an early warning system of financial crises by linking the prominent macroeconomic variables to the movement of the term structure of treasury bonds. The term structure of interest rates is not only the pricing basis of financial assets, but also mirrors the economic and financial conditions of a particular market (Aslanidis and Demiralp, 2020). At a microscopic level, it produces the risk-free benchmark interest rates of loans of different maturities in the market, so as to help investors implement asset pricing and risk management. At the macro level, the term structure of interest rates contains rich information on economic wellness. Practically, the term structure of interest rates reflects the liquidity of the market, which provides a channel for policymakers to transmit price signals to the market. One innovation of the current paper is the exploration of the effectiveness of the term structure of interest rates to serve as an early warning of a financial crisis, an area which has not been well studied so far. The study uses the well-celebrated Diebold-Li model for data fitting as it overcomes the shortcoming of the bootstrap method in the requirement of the number of traded bonds. The model also solves the difficulty relating to spline functions where direct estimation of the forward interest rate beyond the maturity range of coupon-paying bonds is not possible. The Diebold-Li model (Diebold and Li, 2002), as shown in the paper, is superior to other classical models by better capturing the key factors of the term structure of interest rates with highly predictive ability and accuracy. After identification of the key properties of the term structure of interest rates, VAR models were implemented to investigate in what direction and to what extent the early warning indicators of a financial crisis will affect the term structure of treasury bonds. The impulse response and variance decomposition analysis sufficed to validate the proposed scheme. The results of our analysis show that the term structure of treasury bonds plays a critical role in the early warning of financial crises. Signs of financial crisis are analysable according to the change of key factors in the term structure of bonds.

This paper is divided into five sections: Section 1 introduces the background and impact of the method. Section 2 reviews the main related literature in the field, particularly those relating to classical theories and modern models of the term structure of interest rates, and early warning indicators of financial crises. Section 3 focuses on data fitting using the Diebold-Li model, where a two-step method is implemented and proxy variables are constructed to enhance the model fit. In Section 4, we identify the five most predictive early warning indicators of a financial crisis (CPI, loan-to-deposit ratio, M2/M1, real effective exchange rate and P/E ratio) and implement them with a VAR model to explain the dynamic change of term structure of interest rates. In particular, impulse response and variance decomposition analysis are provided to explore in what direction and to what extent the early warning indications of financial crisis may influence the term structure of interest rates. Section 5 summarises the current research with discussions of future directions to perfect such a system in real financial practice.

2 Literature review

The term structure of interest rates is used to describe the relationship between interest rate and term-to-maturity. The yield curve has different shapes, which notably include quasi-flat, increasing, decreasing and humped shapes. Classical theories on the term structure of interest rate have existed since the 19th century, and can be divided into two major categories: pure expectation theory and risk premium theory (including liquidity premium, preferred habitat and market segmentation theories) (Taylor, 1991). The pure expectation theory proposed by Fisher is the cornerstone of classical theories on term structure dynamics. It assumes that the expectation of the future short-term interest rate is the only factor that affects the forward interest rate, where the long-term interest rate is the average of the expected short-term interest rate. Nowadays, pure expectation theory still plays an important role in the theoretical and empirical research of fixed income securities. Longstaff (2000) studied the short-term interest rate and concluded that expectation theory could forecast the short-term and extremely short-term interest rates. Fama (1984) analysed the interest rate of 1–6 months in US and documented evidence disagreeing with the expectation theory. Wu and Xie (2005) used regression and vector autoregression models to verify the term structure of interest rates in China's government bond market. Shi et al. (2005) focused on the analysis of yield curve in different periods and found that before the Asian financial crisis, the CHIBOR rate (China inter-bank offered rate) supported the expectation theory, while after the financial crisis, it could not fully support the expectation theory. Wang and Wang (2010) analysed China's market and came to the conclusion that both the short-term interest rate and the long-term interest rate do not support the expectation theory.

Liquidity premium theory, one of the most widely used theories in the field of fixed income securities, was described by Hicks (1939) and Culbertson (1957). They modified the pure expectation theory by adopting the basic assumption of expectation theory (the expectation of future interest rate will affect the behaviour of investors), but rejected the other assumption of pure expectation theory (investors are indifferent between long-term securities and short-term securities). They introduced risk factors into liquidity premium theory, believing that the liquidity of short-term bonds is higher than long-term bonds. If investors are risk-averse, a preference for highly liquid bonds will drive interest rates lower on short-term bonds than on long-term ones. Horne (1965) concluded that in addition to the expected information, the forward rate should also include risk factors, which serve as the compensation for liquidity. Market segmentation theory was proposed by Culbertson (1957). It regards the bond market with different maturities as different, segmented markets and indicates that the yield of bonds with different maturities depends only on their own supply and demand relationship, not on other bonds. Under the influence of market segmentation theory, Modigliani and Sutch (1966) proposed the preferred habitat theory. They believe that the market is composed of a variety of investors with different preferred investment horizons, and each of them prefers to invest in a specific segment of the yield curve. Hence investors should be compensated if they move away from their preferred segments on the yield curve. If the supply-demand relationship is out of balance, the bond should be sold at a premium or discount on the basis of expected yield. The major drawback of this theory is that it ignores the important link between long-term and short-term bond markets.

Classical theories on term structure of interest rate only describe the shape of and reasons for yield curve movement theoretically, but they cannot examine the specific meaning of yield curve movement from the quantitative perspective, and cannot guide micro investors to manage interest rate risk. Therefore, modern models on the term structure of interest rates mainly focus on the problem of how to fit and forecast the yield curve to price financial assets. The Bootstrap Method is a classical and direct method to derive the term structure of interest rates. It separates the coupons from the principal of a bond and makes them into individual 'zero-coupon bonds'. The interest rate of the corresponding maturity can be derived from the yield of these 'zero-coupon bonds'. This method is relatively simple and suitable for the developed bond market with complete product categories, accurate pricing and strong liquidity. In the underdeveloped bond market, deducing the term structure of interest rates with the bootstrap method may differ greatly from the actual situation. Fama and Bliss (1987) proposed to separate coupons from coupon-paying bonds and calculate the yield to maturity of coupon-paying bonds by a recursive method. However, some studies have shown that while the short-term interest rates fitted by this method are more accurate, fitting the middle-term and long-term rates is not so good. The splines method uses the spline function to fit the discount factor and estimates the parameters by minimising the difference between the theoretical price and the market price of the sample bonds, thus estimating the discount function. Piecewise polynomial, exponential, and spline functions are usually used to fit the discount function. McCulloch (1971) proposed to fit the term structure of interest rate with a piecewise polynomial $f_j(m) = m^j, j = 1, 2 \ldots k$. Langetieg and Smoot (1989) used an exponential discount function $D(t) = \exp(-tR(t))$ to fit the term structure of interest rates, but the parameter estimation was more complex. Shea (1984) first proposed to use a B-spline function $D(t) = \sum_{j=1}^{k} b_j g_j(t)$ (where b_j is the spline parameter, k is the number of spline functions and $g_j(t)$ is the B-spline basis function) to estimate the discount function. Steeley (1991) first used a B-spline model to study the term structure of treasury bonds, and believed that the three-time B-spline model could ensure that the forward interest rate curve was smooth, which was also adopted by many researchers in succession. The Nelson-Siegel model (1987) uses Laguerre functions to construct the bond yield to maturity function with only four estimated parameters, which overcomes the shortcoming that the bootstrap method requires more trading bonds, and it solves the difficulty that the spline function method cannot directly estimate the forward interest rate beyond the maturity range of coupon paying bonds. The formula is as follows:

$$R(0,\theta) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{\theta}{\tau_1})}{\frac{\theta}{\tau_1}} + \beta_2 \left[\frac{1-\exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}} - \exp\left(-\frac{\theta}{\tau_1}\right) \right] \tag{1}$$

where $R(0,\theta)$ is the pure discount rate with maturity θ, β_0 is the level parameter (or the long-term rate), β_1 is the slope parameter (or the spread short/long-term rate), β_2 is the curvature parameter (or middle-term rate) and τ_1 is the scale parameter.

Diebold and Li (2002) put forward a new model, which is conventionally called the DL or DNS model, by studying the dynamic variation of three factors of the Nelson-Siegel model and making dynamic fitting and prediction for the term structure of interest rates. Substantial subsequent empirical research validated the predictive ability of the Diebold-Li model in comparison to classical dynamic models (Sari et al., 2019; Poghosyan and Poghosyan, 2019; Csepregi, 2020). In this paper, we choose the Diebold-Li model for data fitting as it overcomes the shortcoming that the bootstrap method requires more trading bonds. It also solves the difficulty that the spline function method cannot directly estimate the forward interest rate beyond the maturity range of coupon paying bonds. Moreover, the Diebold-Li model is superior to other classical models by capturing the key factors of the term structure of interest rates with higher predictive ability. Additionally, the model is relatively parsimonious and easy to implement with software.

Research in early warning indicators of financial crises began in the 1960s. Haner (1969) designed a country risk index (CRI) for evaluating a country's economic, political, social, environmental risks. The financial indicators include foreign exchange reserves, foreign exchange income, amount of foreign debt, structure of foreign debt, the state of foreign exchange control and government financing ability, etc. Bilson (1979) pointed out that leading indicators are economic indicators that can give an alarm before a crisis occurs. He selected 16 leading indicators of currency crises, all of which will change abnormally when financial risks are concentrated. Reinhart and Rogoff (2008) divided early warning indicators into economic growth, asset price and public debt, and obtained effective early warning indicators through relevant statistical methods, including GDP growth rate per unit of capital, current account/GDP, public debt/GDP, stock price and housing price. Chen et al. (2009) selected early warning indicators of financial crises from four aspects: risk of bubbles, financial market, macroeconomic environment and global economic environment. The early warning indicators with good predictive effect were selected according to the multi-order Granger causality test.

3 Data and term structure fitting with Diebold-Li model

This paper uses the data of spot rates published by China Central Depository and Cleaning Company, which adopted the Hermite interpolation method. After eliminating outliers, this method interpolates the cubic polynomial to obtain the yield curve. This method gives consideration to smoothness, flexibility and stability, and can better reflect the yield curve at different time points. This paper uses daily data on 13 maturities: 1 month, 2 months, 3 months, 6 months, 9 months, 1 year, 3 years, 5 years, 7 years, 10 years, 15 years, 20 years and 30 years. The time span for the data is from January 2007 to December 2019, a total of almost 13 years. All data are publicly and openly accessible at the website of China Central Depository and Cleaning Company at the time when the current research was done.

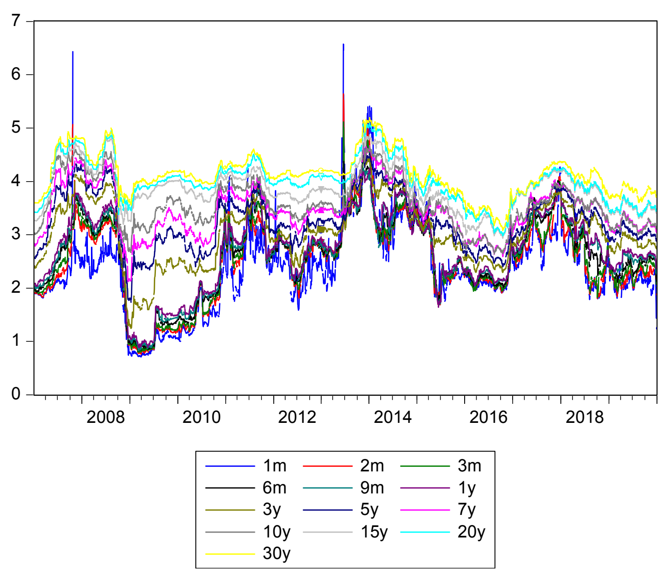
As shown in Table 1, the fluctuation of short-term interest rate is significantly greater than that of long-term interest rate, and the range of short-term interest rate fluctuation is relatively large. The difference between the maximum and minimum values of the one-month spot rate is 5.87%, while the difference between the maximum and minimum values of the 30-year spot rate is only 2.12%. In addition, the average and median interest rates for different maturities meet the law that "the longer the maturity, the higher the interest rate; the shorter the maturity, the lower the interest rate". According to the Jarque-Bera test, the null hypothesis that the distribution is normal is rejected with a 95% confidence level.

Table 1 Descriptive statistics of the data

	1m	*2m*	*3m*	*6m*	*9m*	*1y*	*3y*	*5y*	*7y*	*10y*	*15y*	*20y*	*30y*
Mean	2.38	2.51	2.55	2.63	2.67	2.72	3.06	3.28	3.47	3.59	3.87	4.01	4.14
Median	2.29	2.55	2.59	2.68	2.75	2.80	3.06	3.22	3.44	3.54	3.83	3.99	4.12
Maximum	6.58	5.64	5.11	4.37	4.25	4.25	4.50	4.53	4.67	4.72	4.91	5.10	5.20
Minimum	0.71	0.76	0.80	0.82	0.83	0.89	1.24	1.73	2.12	2.64	2.94	2.98	3.08
Std. Dev	0.82	0.77	0.76	0.73	0.72	0.72	0.60	0.52	0.47	0.46	0.41	0.42	0.40
Skewness	0.56	− 0.04	− 0.20	− 0.39	− 0.47	− 0.49	− 0.26	0.09	0.18	0.34	0.29	0.25	0.25
Kurtosis	4.44	3.24	2.99	2.80	2.81	2.79	2.87	2.50	2.54	2.52	2.82	3.02	3.12
Jarque-Bera	446.86	8.33	21.95	88.63	122.47	136.40	39.12	38.17	46.18	93.37	49.40	33.41	36.21
P value	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

It can be seen from Figure 1 that since 2007, China's bond market has experienced several obvious swings between bull and bear markets. From 2007 to the second quarter of 2008, the yield curve moved up on the whole. After that, it entered a great bull market for nearly a year, with yields close to historic lows. After the third quarter of 2009, the yield curve moved up. The bond market re-entered a bull market in the third quarter of 2011. Since the third quarter of 2012, the yield curve moved up slowly and entered a bear market. The bond market entered a bull market again in the fourth quarter of 2013, and then entered a bear market in the third quarter of 2016. After 2018, the bond market entered a bull market again.

Figure 1 Interest rates with different maturities (see online version for colours)



In addition, with more than a dozen maturities, it is difficult for us to use them all in empirical analysis at the same time. Therefore, we need to extract important factors from the term structure of interest rates to facilitate subsequent analysis. In statistics, principal component analysis (PCA) is usually used to extract yield curve features to reduce dimensions, so as to obtain the principal components that can explain the variance of the yield curve. However, as yield curves at different time points usually change in the same direction (especially for those yield curves with very close maturities), the data is likely to be non-stationary. Therefore, the applicability of the PCA method is still questionable. Another method to fit the yield curve is the Nelson-Siegel model or its extended models. Those models not only have a strong theoretical basis, but also have a wide range of application. Therefore, this paper uses the Diebold-Li model (which is the re-parameterised Nelson-Siegel model) to fit the term structure of Treasury bonds. The formula is as follows:

$$y_t(\tau) = L_t + S_t\left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau}\right) + C_t\left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right) \tag{2}$$

where t is the observation date; τ is the time to maturity; $y_t(\tau)$ is the observed yield; L_t is the level factor (or long-term rate); S_t is the slope factor (or short-term rate); C_t is the curvature factor (or medium-term rate); and λ is a parameter which determines the maturity at which the loading on the curvature is maximised, and governs the exponential decay rate of the model.

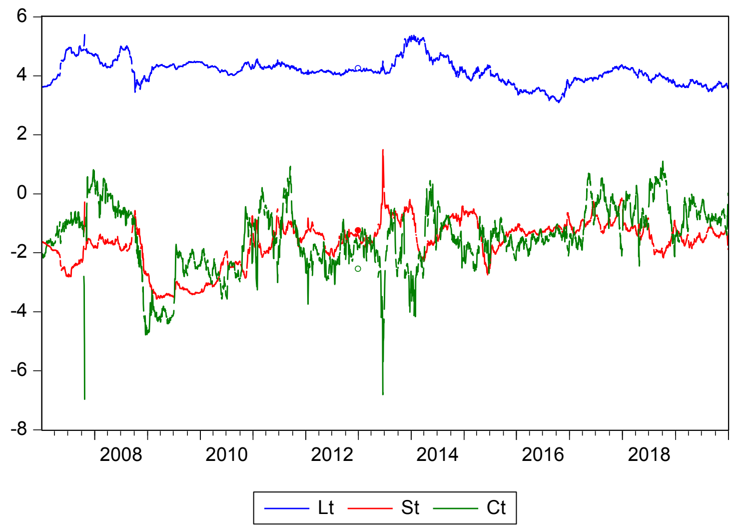
We estimate the parameters using a two-step approach. First, it is customary to set $\lambda = 0.0609$, which means that the loading on the curvature is maximised at 30 months. Then, we use the ordinary least square (OLS) method to estimate level, slope, and curvature factors. This OLS is repeated for all observed yield curves to accumulate a 3D time series of estimates. Then we fit all 3 factors to a VAR (1) model simultaneously.

As shown in Table 2, the mean value of L_t is 4.1527%. As a level factor, L_t represents the level of long-term interest rates, indicating that the long-term interest rate of Treasury bonds is about 4.1527%. The mean value of slope factor S_t is –1.6423%, which represents the difference between short-term and long-term interest rates. $L_t + S_t$ represents the short-term interest rate. According to Table 2, the sum of the two is 2.5104%, indicating that the short-term interest rate of treasury bonds is about 2.5104%. C_t is the curvature factor whose mean value is –1.4925. As can be seen from Figure 2, L_t has the most stable trend with the least fluctuation, indicating that the long-term interest rate of treasury bonds is relatively stable. The curvature factor C_t fluctuates the most.

Table 2 Parameter estimates for DL model

	L_t	S_t	C_t
Mean	4.1527	–1.6423	–1.4925
Median	4.1571	–1.4885	–1.4561
Maximum	5.3975	1.4999	1.1018
Minimum	3.0928	–3.5696	–6.9688
Std. Dev	0.4165	0.7556	1.0961
Skewness	0.2569	–0.7350	–0.5800
Kurtosis	3.2462	3.4221	3.7623

Figure 2 Parameter Estimates (graph) (see online version for colours)



From Table 3, it can be seen that L_t, S_t and C_t are correlated, and S_t and C_t are positively correlated, indicating that the difference between long-term and short-term interest rates and the curvature have a certain positive relationship. In addition, it also shows that the three factors affect each other.

Table 3 Correlation analysis

	L_t	S_t	C_t
L_t	1		
S_t	–0.208570	1	
C_t	–0.079057	0.321609	1

Following Diebold et al. (2006), we construct proxy variables by using market short-term (6 months), medium-term (5 years) and long-term (30 years) spot rates to test whether L_t, S_t and C_t factors can well represent the shape of the term structure of interest rates.

The proxy variable of level factor： $Proxy\ L_t = \dfrac{R(6m) + R(3y) + R(20y)}{3}$

The proxy variable of slope factor： $Proxy\ S_t = R(6m) - R(20y)$

The proxy variable of curvature factor： $Proxy\ C_t = 2R(3y) - R(6m) - R(20y)$

The correlation coefficient between proxy L_t and L_t, proxy S_t and S_t proxy C_t and C_t are 0.618475, 0.968113 and 0.796468 respectively. It can be seen from Figures 3–6 that the trend of L_t, S_t and C_t and the corresponding proxy variables is relatively consistent and highly correlated, so it is feasible to describe the term structure of interest rates by using L_t, S_t and C_t.

Figure 3 Comparison between proxy L_t and L_t (see online version for colours)

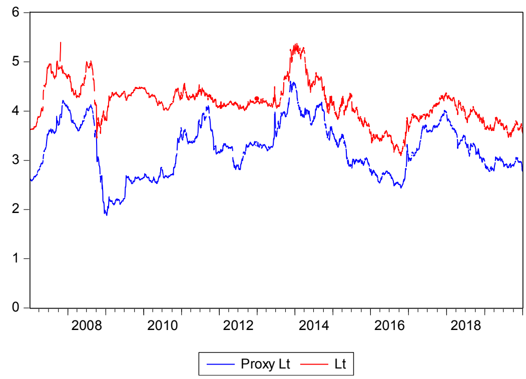


Figure 4 Comparison between proxy S_t and S_t (see online version for colours)

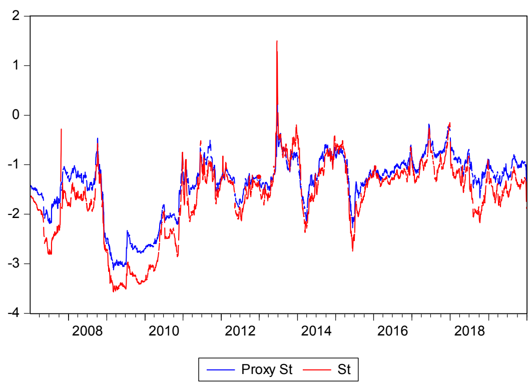


Figure 5 Comparison between proxy C_t and C_t (see online version for colours)

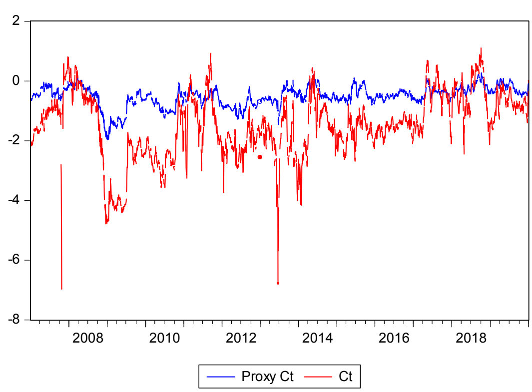
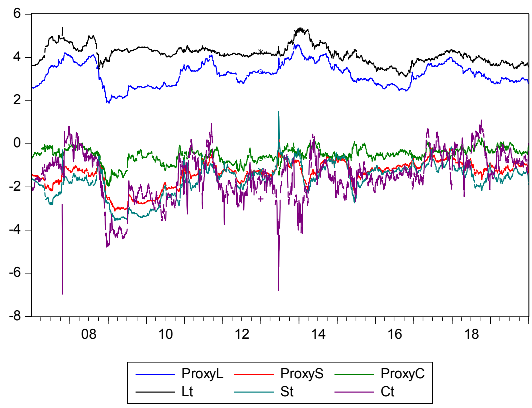


Figure 6 Comparison between proxies and yield curve factors (see online version for colours)



4 Empirical analysis

The design and selection of early warning indicators are critically related to the success of financial crisis prediction. Based on the survey of references on early warning indicator system of financial crises at home and abroad (Samiras et al., 2020; Wang et al., 2020; Klopotan et al., 2018), this paper divides the early warning indicators of financial crises into the following three categories. The first category is macroeconomic indicators, including GDP growth rate, CPI, growth rate of industrial added value, rate of export change, etc. The second category involves performance and operations of financial system, including the following indicators: domestic credit/GDP, real interest rate, M2/GDP, M2 multiplier (M2/M1), loan-to-deposit ratio, savings deposit/M2, Banks' foreign currency liabilities/GDP, growth rate of state foreign exchange reserves, M2/foreign exchange reserve, real effective exchange rate and foreign direct investment/GDP, etc. Lastly, financial bubble indicators are included: stock price/earnings ratio, securitisation ratio (market value of shares/GDP), change in rates of house price index and construction loan/bank loan, etc.

Although the number of indicators listed above is large, many of them are highly correlated. In addition, limitations such as data availability and comparability (between different countries and periods) often make it unrealistic for some indicators to be included in the early warning system. In general, 1–3 indicators are selected from each category based on the content. Moreover, the results show that the quality of the early warning system of financial crises is not highly correlated with the quantity of indicators, but rather depends on the quality of the selected indicators and whether they can convey more valuable information. In summary of the above considerations, this paper selects the following five early warning indicators and further studies their effects on the term structure of Treasury bonds.

1 *CPI*: It is a significant indicator measuring inflation, which is a general and sustained rise in the price level. In addition, CPI is also a significant indicator measuring the effectiveness of government policies.

2 *Loan-to-deposit ratio*: It is one of the indicators measuring the liquidity risk of banks. The higher the ratio is, the more loan assets correspond to liabilities, the lower

the liquidity of the bank, and the more vulnerable the banking system as banks do not have sufficient current assets to cope with external shocks.

3 *M2 multiplier (M2/M1)*: An increase in the monetary multiplier means that the monetary base is amplified by monetary derivatives and the economic bubble is larger.

4 *Real effective exchange rate*: It reflects the stability of currency value and the exchange rate risk. External vulnerability and currency overvaluation will increase the vulnerability of the banking industry, because the loss of competitiveness in the external market will lead to economic recession, business failure and decline in loan quality. A banking crisis can lead to a currency crisis.

5 *Price/Earnings ratio (Shanghai Main Board Market P/E Ratio)*: The accumulation and release of financial risks will be reflected through the stock market, and the outstanding manifestation is the abnormal fluctuation of asset prices. If asset prices fall sharply, financial assets will shrink and financial asset bubbles will burst, forming a large number of non-performing assets at financial institutions, worsening the economy, and triggering a financial crisis.

This paper selects monthly data from 2007 to 2019 for empirical analysis. The data are from the official websites of the National Bureau of Statistics, the Shanghai Stock Exchange, the People's Bank of China and the Bank for International Settlements. In order to eliminate the seasonal effect, the growth rate data used in this paper are all year-on-year growth rates; that is, the percentage change of each variable over the same period of last year. In addition, in order to maintain the consistency of the data, the daily data used in Section 3 are converted into monthly data. Before building the VAR model, we need to test the stationarity of the time series so as to avoid spurious regression. This paper uses the ADF test for stationarity, the results of which are as follows:

As can be seen from Table 4, the series of L_t, S_t and C_t are stationary at the significance level of 10%. The other five series are not stationary in themselves, but are stationary in the first-order difference. Therefore, the first-order differenced data are used for these five series.

Table 4 ADF test results

	Level			*1st difference*		
	t-Statistic	*Prob.**	*Significance level of 10%*	*t-Statistic*	*Prob.**	*Significance level of 10%*
L_t	–3.0240	0.0349	Stationary	–	–	–
S_t	–2.8154	0.0584	Stationary	–	–	–
C_t	–4.5089	0.0003	Stationary	–	–	–
CPI	–2.4944	0.1188	Non-stationary	–8.9445	0.0000	Stationary
Loan-to-deposit ratio	–1.6236	0.7793	Non-stationary	–6.4124	0.0000	Stationary
M2/M1	–1.5731	0.4939	Non-stationary	–13.3082	0.0000	Stationary
Real effective exchange rate	–1.7952	0.3818	Non-stationary	–7.7887	0.0000	Stationary
Market P/E ratio	–2.0679	0.2581	Non-stationary	–5.7865	0.0000	Stationary

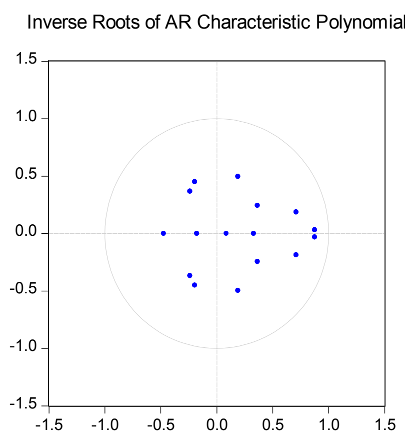
Next, we use L_t, S_t, C_t, DCPI (CPI after the first-order difference), DLDR (Loan-to-deposit Ratio after the first-order difference), DM2/M1(M2/M1 after the first-order difference), DREER (Real Effective Exchange Rate after the first-order difference) and DP/E (Shanghai Main Board Market P/E Ratio after the first-order difference) to build our VAR (p) model. Through repeated comparison of information criteria such as AIC and SC as shown in Table 5, it is found that the VAR (2) model is most suitable.

Table 5 VAR lag order selection criteria

Lag	*LogL*	*LR*	*FPE*	*AIC*	*SC*	*HQ*
0	–1107.300	NA	0.000537	15.17415	15.33689	15.24027
1	–683.3351	796.0156	4.02e–06	10.27667	11.74137*	10.87179*
2	–604.1481	140.0587	3.29e–06*	10.07004*	12.83670	11.19417
3	–547.2667	94.41533	3.69e–06	10.16689	14.23551	11.82002
4	–497.2274	77.61201	4.63e–06	10.35684	15.72741	12.53896
5	–432.2117	93.76419	4.84e–06	10.34302	17.01555	13.05414
6	–379.5418	70.22655	6.18e–06	10.49717	18.47165	13.73729
7	–302.4842	94.35616*	5.90e–06	10.31951	19.59596	14.08863
8	–238.3207	71.58386	7.06e–06	10.31729	20.89569	14.61541

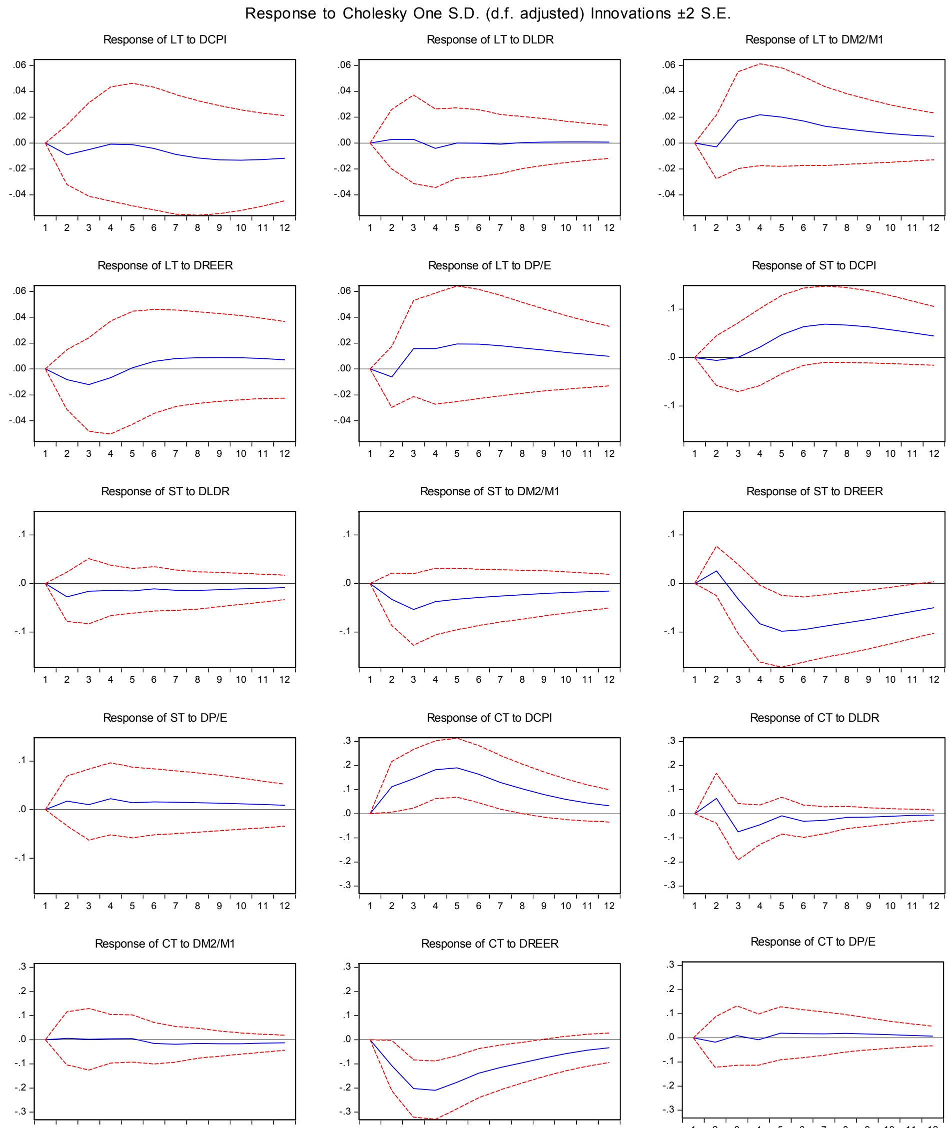
The vector autoregression estimates of the VAR (2) model are shown in the Appendix Table A1. As can be seen from that table, the estimates are significant at the 90% confidence level, and the R-square value of L_t, S_t, C_t are 0.887952, 0.843516 and 0.672894 respectively. We also need to determine the stationarity of the VAR (2) model. Only a stationary VAR model can carry out subsequent impulse response and variance decomposition. As can be seen from Figure 7, the reciprocal absolute values of the characteristic roots are all less than 1, and all the reciprocal values of the characteristic roots are within the unit circle, indicating that the model is stationary.

Figure 7 Unit root test (see online version for colours)



The VAR model does not let us directly observe the effect of early warning indicators of financial crises on the factors L_t, S_t, C_t. Our purpose is to know in what direction these indicators effect the factors L_t, S_t, C_t and how much the effect can be, so we need to use the impulse response and variance decomposition method. We conduct the impulse responses with 12 periods, the results of which are shown in Figure 8.

Figure 8 Impulse responses and the 0.95 confidence levels (see online version for colours)



As can be seen from Figures 8 and 9 and Table 6, the impact of DCPI, DLDR, DM2/M1, DREER and DP/E on L_t, S_t, C_t varies from each other. L_t has the largest response to DP/E, which reaches the maximum in the fourth period, and the smallest response to

DLDR, which reaches the minimum in the fifth period. When DCPI gives L_t a one-unit positive impact, L_t always responds negatively, and reaches the maximum response in the tenth period. When DLDR gives L_t a one-unit positive impact, L_t responds positively at first, then responds negatively and reaches the maximum response in the fourth period. Only a weak negative response exists in the twelfth period. When DM2/M1 gives L_t a one-unit positive impact, L_t responds negatively at first, then responds positively in the third stage, and reaches the maximum degree of the positive response in the fourth period. When DREER gives L_t a one-unit positive impact, L_t responds negatively at first and reaches the maximum degree of response in the third period, and then responds positively in the fifth period. When DP/E gives L_t a one-unit positive impact, L_t responds negatively at first, then responds positively in the third period, and reaches the maximum degree of the positive response in the fifth period.

Table 6 Impulse responses (table)

Response of LT					
Period	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
1	0.000000	0.000000	0.000000	0.000000	0.000000
2	–0.009036	0.002972	–0.002992	–0.008230	–0.006069
3	–0.005006	0.002956	0.017606	–0.012010	0.015801
4	–0.000798	–0.004087	0.021871	–0.006606	0.015723
5	–0.001221	1.59E–05	0.019982	0.000925	0.019486
6	–0.004272	–0.000134	0.016966	0.005863	0.019361
7	–0.008773	–0.000829	0.013078	0.008243	0.018075
8	–0.011501	0.000353	0.010848	0.008793	0.016337
9	–0.012867	0.000846	0.008972	0.008934	0.014699
10	–0.013215	0.000854	0.007284	0.008759	0.012855
11	–0.012789	0.000927	0.006116	0.008120	0.011315
12	–0.011812	0.000845	0.005204	0.007130	0.009950
Response of ST					
1	0.000000	0.000000	0.000000	0.000000	0.000000
2	–0.006189	–0.027307	–0.032772	0.025941	0.017734
3	0.000538	–0.015928	–0.053563	–0.031948	0.010256
4	0.021490	–0.014384	–0.037348	–0.083008	0.022355
5	0.047414	–0.015150	–0.032218	–0.098512	0.014619
6	0.063532	–0.011056	–0.028745	–0.095094	0.016180
7	0.068815	–0.013841	–0.025746	–0.087689	0.015185
8	0.067036	–0.014411	–0.023287	–0.080988	0.014517
9	0.063102	–0.012457	–0.020386	–0.074287	0.013569
10	0.057541	–0.011014	–0.018593	–0.066255	0.012281
11	0.051089	–0.009643	–0.017188	–0.057723	0.010588
12	0.044428	–0.008210	–0.015619	–0.049660	0.009037

Table 6 Impulse responses (table) (continued)

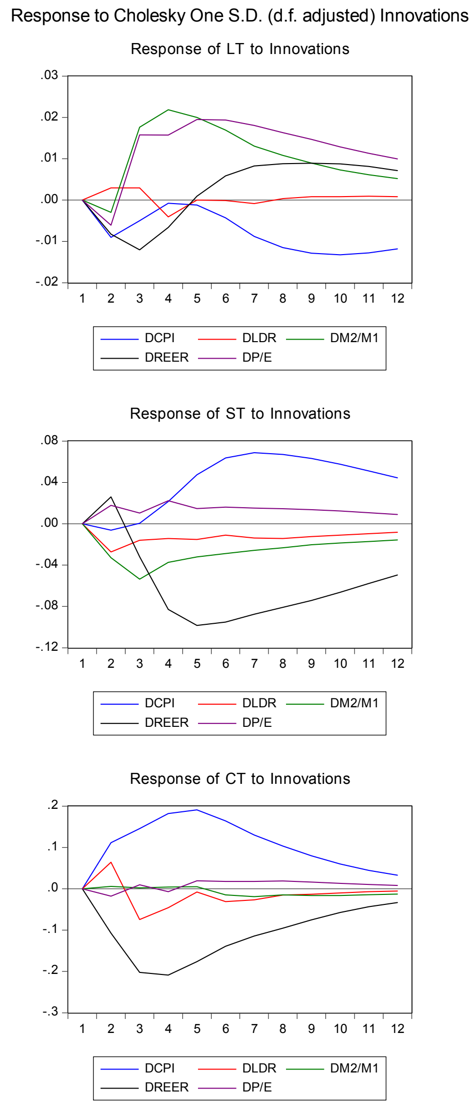
Response of CT					
Period	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
1	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.111571	0.063919	0.005611	–0.107427	–0.017808
3	0.145258	–0.074528	0.001839	–0.202480	0.009352
4	0.181949	–0.045966	0.003944	–0.209137	–0.007140
5	0.190662	–0.008145	0.004769	–0.176488	0.018902
6	0.163807	–0.031290	–0.014874	–0.139147	0.017363
7	0.129677	–0.027022	–0.019241	–0.114725	0.017339
8	0.103052	–0.015524	–0.014893	–0.095559	0.018645
9	0.079523	–0.013558	–0.016359	–0.075455	0.016252
10	0.059967	–0.010421	–0.016472	–0.057695	0.012873
11	0.044560	–0.007108	–0.014596	–0.043787	0.010447
12	0.032759	–0.005568	–0.013105	–0.033409	0.007881

S_t has the largest response to DREER, which reaches the maximum in the fifth period, and the smallest response to DP/E, which reaches the minimum in the twelfth period. When DCPI gives S_t a one-unit positive impact, S_t responds negatively at first, then responds positively in the third stage, and reaches the maximum degree of the positive response in the seventh period. When DLDR gives S_t a one-unit positive impact, S_t always responds negatively, and reaches the maximum degree of response in the second period. When DM2/M1 gives S_t a one-unit positive impact, S_t always responds negatively, and reaches the maximum degree of response in the third period. When DREER gives S_t a one-unit positive impact, S_t responds positively at first, then responds negatively in the third period, and reaches the maximum degree of response in the fifth period. When DP/E gives S_t a one-unit positive impact, S_t always responds positively, and reaches the maximum degree of response in the fourth period.

C_t has the largest response to DREER, which reaches the maximum in the fourth period, and the smallest response to DP/E, which reaches the minimum in the fourth period. When DCPI gives C_t a one-unit positive impact, C_t always responds positively, and the degree of response reaches the maximum in the fifth period. When DLDR gives C_t a one-unit positive impact, C_t responds positively at first, then it responds negatively and reaches the maximum degree of response in the third period. When DM2/M1 gives C_t a one-unit positive impact, C_t responds positively at first, then responds negatively in the sixth period. It reaches the maximum degree of response in the seventh period. When DREER gives C_t a one-unit positive impact, C_t always responds negatively and reaches the maximum degree of response in the fourth period. When DP/E gives C_t one-unit positive impact, C_t responds negatively at first, then it rises and falls in the first five periods, and reaches the maximum degree of the positive response in the fifth period.

Finally, we carry out the variance decomposition for the VAR (2) model, and the results are shown in Table 7.

Figure 9 Impulse responses (combined graphs) (see online version for colours)



The largest contribution to the variance of L_t is always L_t itself, which contributes 100% in the first period and decreases gradually after that. However, it still contributes 94.18865% until the twelfth period, far higher than the contribution degree of any other variable. Among the five early warning indicators, the contribution of DP/E is the highest. It shows an upward trend in each period, and reaches the maximum value of 1.320472% in the twelfth period. DLDR has the lowest contribution degree, and its contribution degree is the highest in the fourth period, with the maximum value being only 0.031563%. The largest contribution to the variance of S_t is always S_t itself, which contributes 95.42593% in the first period and decreases gradually after that. However, it still contributes 66.91858% until the twelfth period, far higher than the contribution degree of any other variable. Among the five early warning indicators, the contribution of

DREER is the highest. It shows an upward trend in each period, and reaches the maximum value of 9.414752% in the twelfth period. DP/E has the lowest contribution degree, and its contribution degree is the highest in the twelfth period, with the maximum value being only 0.388671%. The largest contribution to the variance of C_t is always C_t itself, which contributes 71.06433% in the first period. It still contributes 54.97702% in the twelfth period, far higher than the contribution degree of any other variable. Among the five early warning indicators, the contribution of DREER is the highest. It shows an upward trend in each period and reaches the maximum value of 14.44413% in the twelfth period. DM2/M1 has the lowest contribution degree, and its contribution degree is the highest in the twelfth period, with the maximum value being only 0.144342%. Additionally, we can find that the five early warning indicators have the largest impact on the curvature factor C_t, followed by the slope factor S_t. Therefore, we may find signs of a financial crisis by analysing the change of the slope factor S_t and curvature factor C_t.

Table 7 Variance decomposition

				Variance decomposition of LT					
Period	*S.E.*	*LT*	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
1	0.144912	100.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.227552	98.65013	0.304369	0.651514	0.157679	0.017064	0.017290	0.130817	0.071135
3	0.287237	98.04632	0.281152	0.531188	0.129330	0.021303	0.386542	0.256917	0.347243
4	0.329548	97.78470	0.220041	0.403952	0.098839	0.031563	0.734113	0.235367	0.491429
5	0.359125	97.43627	0.219734	0.398174	0.084384	0.026578	0.927764	0.198858	0.708238
6	0.380888	96.90937	0.346127	0.521590	0.087599	0.023640	1.023192	0.200481	0.888006
7	0.396979	96.32061	0.535232	0.689548	0.129482	0.022199	1.050460	0.227670	1.024798
8	0.409050	95.74493	0.728982	0.859035	0.201006	0.020982	1.059709	0.260634	1.124721
9	0.418169	95.23238	0.904248	1.001053	0.287009	0.020487	1.060028	0.295037	1.199760
10	0.425077	94.80384	1.057494	1.108300	0.374400	0.020230	1.055221	0.327986	1.252534
11	0.430307	94.45796	1.188652	1.182486	0.453680	0.020205	1.049928	0.355667	1.291420
12	0.434267	94.18865	1.300177	1.229663	0.519425	0.020217	1.045226	0.376166	1.320472
				Variance decomposition of ST					
1	0.317765	4.574074	95.42593	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.449315	5.536106	91.86268	1.191774	0.018975	0.369370	0.531986	0.333318	0.155788
3	0.531395	5.601884	88.54126	3.344554	0.013668	0.353917	1.396333	0.599758	0.148631
4	0.589606	5.304508	83.74054	6.194780	0.143952	0.346997	1.535480	2.469257	0.264482
5	0.635368	4.855401	78.97916	8.738534	0.680844	0.355665	1.579388	4.530312	0.280694
6	0.672868	4.468079	75.23981	10.51379	1.498581	0.344126	1.590758	6.036748	0.308104
7	0.703370	4.168270	72.54173	11.61063	2.328616	0.353648	1.589764	7.078777	0.328568
8	0.727646	3.959670	70.58030	12.27797	3.024581	0.369666	1.587882	7.853118	0.346810
9	0.746566	3.820908	69.15612	12.66062	3.587637	0.379007	1.582984	8.450237	0.362487
10	0.761047	3.734693	68.13672	12.87139	4.024057	0.385664	1.583000	8.889616	0.374864
11	0.771947	3.688444	67.42086	12.98016	4.349225	0.390455	1.588188	9.199503	0.383166
12	0.780046	3.672783	66.91858	13.03250	4.583777	0.393466	1.595471	9.414752	0.388671

Table 7 Variance decomposition (continued)

Variance decomposition of CT									
Period	*S.E.*	*LT*	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
1	0.643582	13.30133	15.63435	71.06433	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.761168	10.12741	11.37862	73.58821	2.148515	0.705189	0.005433	1.991891	0.054737
3	0.870301	7.775774	8.769348	70.75841	4.429203	1.272741	0.004602	6.936502	0.053417
4	0.954436	6.468885	7.544207	66.75537	7.316921	1.290189	0.005534	10.56889	0.050012
5	1.012850	5.763487	7.408074	63.12790	10.04082	1.152129	0.007132	12.42122	0.079239
6	1.052192	5.373083	7.997213	60.36019	11.72767	1.156017	0.026592	13.25858	0.100656
7	1.078104	5.128881	8.726215	58.42328	12.61748	1.163936	0.057180	13.76128	0.121741
8	1.095015	4.973119	9.360307	57.07975	13.11645	1.148363	0.073926	14.10108	0.147002
9	1.105875	4.876195	9.872087	56.18247	13.37719	1.140950	0.094363	14.29102	0.165726
10	1.112739	4.820894	10.26656	55.59748	13.50309	1.135689	0.115115	14.38409	0.177071
11	1.117070	4.796098	10.55251	55.22056	13.55772	1.130949	0.131297	14.42642	0.184447
12	1.119841	4.792633	10.74926	54.97702	13.57629	1.127831	0.144342	14.44413	0.188488

5 Discussion

The economics factors are often correlated with each other; thus it is unrealistic to assume the independence of the error term between different factors in the VAR model. For example, an increased average income may drive the consumption to a higher level. Also, the fluctuations of stock markets (hence the change of price to earnings ratio) are proved to be correlated with the fluctuation of CPI (Cai et al., 2009). Therefore, further insights could be achieved when correlations of the error term between factors are added into consideration in model diagnostics, the purpose of which is usually accomplished with the orthogonalised impulse response and variance decomposition. However, this method is known to be sensitive to the ordering of the factors given. To illustrate the reasoning for the sensitivity analysis, we follow the presentation similar to Lütkepohl (2005). Assume we are considering a VAR(n) model with p factors, or equivalently, a recursive relation for $X_t = \left(X_{t1}, X_{t2}, X_{t3}, \dots, X_{tp}\right)$ where

$$X_t = \sum_{i=1}^{n} A_i X_{t-i} + \epsilon_t \quad \epsilon_t \sim WN\left(0, \Sigma_\epsilon\right) \tag{3}$$

The positive definite property of the covariance matrix allows us to perform the Cholesky decomposition, which yields $\Sigma_\epsilon = LL^T$. We further denote $diag\left(L\right) = \Lambda$ to be the diagonal of the lower triangular matrix of L. Multiplying to the both sides of the X_t equation of by ΛL^{-1} results in

$$X_t = \left(I_p - \Lambda L^{-1}\right) X_t + \Lambda L^{-1} \sum_{i=1}^{n} A_i X_{t-i} + v_t$$

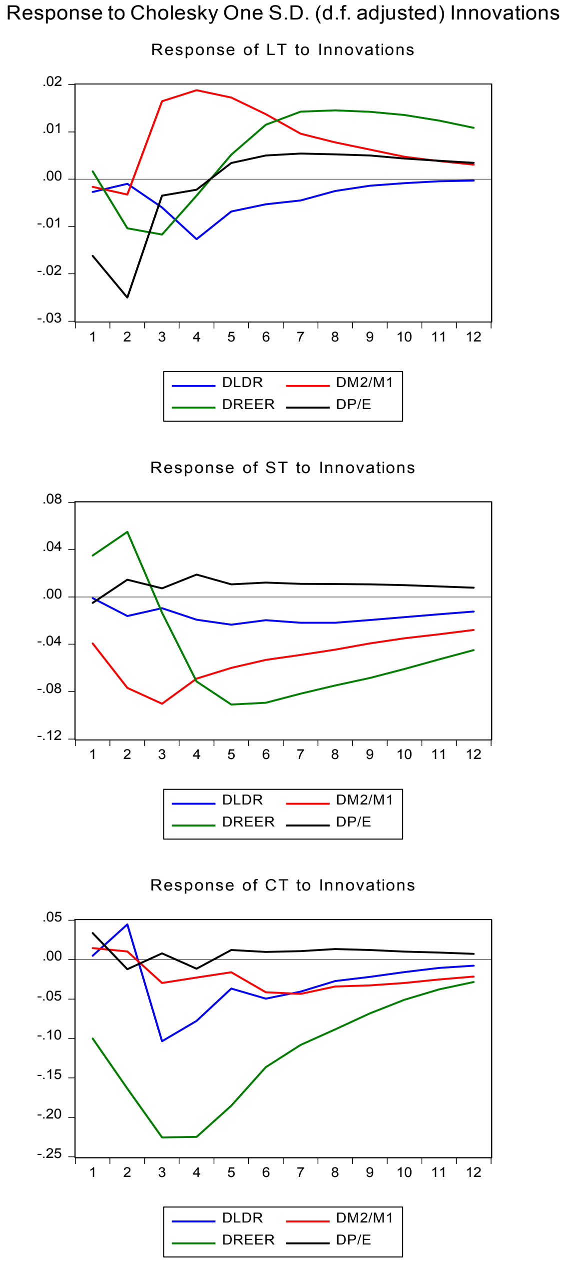
where the covariance of v_t turns out to be $\sigma\left(v_t\right) = \sigma\left(\Lambda L^{-1}\epsilon_t\right) = \Lambda L^{-1}\Sigma_\epsilon \left(L^T\right)^{-1} \Lambda = \Lambda^2$.

Therefore, v_t is the uncorrelated noise that we wish to use as the economic shock in our experiments. Noticing that $\left(I_p - \Lambda L^{-1}\right)$ is a lower triangular matrix with 0 on its diagonal term, a shock in the term X_{ts} will not have the instant current time impact for X_{ti} with $i < s$. Therefore, X_{t1} should be the term which has the immediate effect to all term $X_{t2}, X_{t3}, \ldots, X_{tp}$ when it shocks. Follow this logic, X_{t2} should be the term which has spontaneous effect on $X_{t3}, \ldots, X_{tp}$. In summary, the ordering of the factors is important to impulse response results and the variance decomposition. As it is hard to determine the order precisely, one may switch the order of factors in practice to test if the results are sensitive or invariant to the ordering.

The Cholesky ordering used in our empirical analysis section is, by default, LT, ST, CT, DCPI, DM2/M1, DREER. We now change the ordering so that the new Cholesky ordering for the experiment is DCPI, DM2/M1, DREER, DLDR, DPE, ST, CT, LT, reflecting our conjecture of the chain of the economic force transmission under our model. We therefore perform the impulsive response and variance decomposition using this ordering and results are shown in Figure 10 and Table 8, respectively.

It is shown from the impulsive response that the behaviours of DCPI, DLDR, DP/E change significantly in its pattern when we look at the response of LT, while the response pattern of the ST and CT does not vary significantly. Therefore, the results that we can still conclude are the shocks of DM2/M1 and DREER for the LT component of the yield curve while we should remain cautious for the other shocks which are sensitive to the ordering. For the short term and median term rate, all the impulse response remains relatively valid. The positive shock of DM2/M1 will drive down the long-term rate for a short period of time because the market is experiencing a booming business activity and investment and the expectation for the long-term economy is optimistic with high M2. However, due to the lack of consumption, the market soon realises that the increase in the investment is not proportional to the market consumption and demand with low M1. Hence there is a risk of economic bubble, and the long-term rate is driven up quickly. The influence of a positive shock in the exchange rate with respect to the rate might be explained by the interest rate parity partially, that is, $\left(1+i\right) FR = REER\ (1+i_f)$, where FR represents for forward exchange rate, i_f represents for foreign interest rate. Therefore, there is a positive response of short-term rate with respect to the shock of the DREER. However, the economic reasons behind the long-term change and the change of long term rate after the shock of exchange rate is rather mixed. Therefore, we suggest a more careful examination in real practice.

Figure 10 Impulse responses (table) with Cholesky ordering of DCPI, DM2M1, DREER, DLDR, DPE, ST, CT, and LT (see online version for colours)



Another apparent pattern from the impulse response is that the surge of CPI will drive up the short-term and long-term rate to a large extent because the lenders are expecting a higher risk-free rate to compensate for the high inflation. Additionally, the impulse response of the short term rate results showed that there is a negative relation between the stock market shock (P/E) and the short term rate. The results of such negative correlation between the stock market and the bond market are consistently empirically proved in previous studies, for example Baz et al. (2019). For the variance decomposition part, the results remain largely the same except for the long-term part. However, we can see from Table 8 by comparing to Table 7 that DM2/M1 remains a good predictive factor for the long-term rate while we are not sure of the validity of DP/E to predict LT after sensitivity analysis. DCPI and DREER remain the best indicators for ST and CT, as it is invariant to the Cholesky ordering. Therefore, we are more confident to use it as prediction.

Table 8 Variance decomposition with Cholesky ordering of DCPI, DM2M1, and DREER

				Variance decomposition of LT					
Period	*S.E.*	*LT*	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
1	0.144912	73.78169	4.882159	19.98622	0.041658	0.035443	0.012436	0.012897	1.247500
2	0.227552	75.87091	6.777522	15.31175	0.069003	0.016096	0.025431	0.213077	1.716210
3	0.287237	76.22376	6.468573	15.47279	0.045298	0.053581	0.345045	0.299150	1.091798
4	0.329548	75.66508	6.004462	16.44164	0.040184	0.188186	0.587939	0.238636	0.833879
5	0.359125	74.91707	5.432271	17.76174	0.035518	0.194674	0.725711	0.221756	0.711264
6	0.380888	74.00705	4.918984	19.13449	0.034650	0.192411	0.774864	0.288134	0.649410
7	0.396979	73.14772	4.545108	20.27061	0.063207	0.189873	0.772096	0.394883	0.616503
8	0.409050	72.39148	4.282140	21.16427	0.120008	0.182613	0.763463	0.498905	0.597123
9	0.418169	71.77677	4.097600	21.82530	0.192611	0.175829	0.752729	0.593500	0.585656
10	0.425077	71.30207	3.967412	22.29672	0.268797	0.170548	0.740998	0.676119	0.577342
11	0.430307	70.95168	3.875574	22.62212	0.339206	0.166530	0.730917	0.742403	0.571577
12	0.434267	70.70469	3.811074	22.84084	0.398216	0.163559	0.722600	0.791524	0.567497
				Variance decomposition of ST					
1	0.317765	0.000000	97.19411	0.000000	0.056531	0.001179	1.512465	1.210809	0.024910
2	0.449315	0.045922	92.44102	1.355939	0.120458	0.130481	3.681278	2.105181	0.119717
3	0.531395	0.255501	89.42818	2.905459	0.097549	0.124805	5.517489	1.566725	0.104292
4	0.589606	0.720912	85.56735	4.570224	0.159440	0.206872	5.846548	2.739941	0.188711
5	0.635368	1.292014	81.37517	5.880346	0.610333	0.313760	5.925283	4.411762	0.191327
6	0.672868	1.788630	77.98180	6.706028	1.348678	0.365433	5.908519	5.697827	0.203088
7	0.703370	2.152064	75.48662	7.156699	2.113653	0.431230	5.890634	6.558025	0.211070
8	0.727646	2.371722	73.68071	7.412537	2.759267	0.493087	5.877981	7.184679	0.220019
9	0.746566	2.487041	72.39422	7.547348	3.283270	0.536327	5.857784	7.664782	0.229223
10	0.761047	2.535710	71.49101	7.618561	3.689502	0.566696	5.848229	8.012687	0.237606
11	0.771947	2.544384	70.87239	7.658004	3.991428	0.587038	5.850143	8.252591	0.244024
12	0.780046	2.531733	70.45392	7.684315	4.208227	0.599868	5.857051	8.415789	0.249096

Table 8 Variance decomposition with Cholesky ordering of DCPI, DM2M1, and DREER (continued)

Variance decomposition of CT									
Period	*S.E.*	*LT*	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
1	0.643582	0.000000	8.497158	88.74292	0.000691	0.006024	0.052512	2.425803	0.274891
2	0.761168	1.882845	6.074948	82.90035	2.163166	0.349364	0.055845	6.351314	0.222163
3	0.870301	3.840404	4.976937	73.14902	4.442927	1.681498	0.160514	11.57061	0.178092
4	0.954436	4.961674	4.714654	65.41829	7.320133	2.063168	0.191085	15.16801	0.162986
5	1.012850	5.663335	5.255864	59.94415	10.00796	1.964656	0.195241	16.80976	0.159035
6	1.052192	6.103087	6.187675	56.26871	11.65223	2.041564	0.336713	17.25432	0.155700
7	1.078104	6.282496	7.129166	53.91245	12.50563	2.085739	0.482510	17.44392	0.158082
8	1.095015	6.327427	7.925218	52.39233	12.97683	2.083460	0.564964	17.56182	0.167949
9	1.105875	6.314289	8.543442	51.42876	13.21759	2.081208	0.641311	17.59650	0.176894
10	1.112739	6.279076	9.012195	50.82741	13.32948	2.075583	0.705426	17.58780	0.183026
11	1.117070	6.242301	9.355879	50.45370	13.37459	2.068575	0.751075	17.56599	0.187896
12	1.119841	6.212907	9.597916	50.21976	13.38692	2.063123	0.784803	17.54345	0.191123
Cholesky One S.D. (d.f. adjusted)									
Cholesky ordering: DCPI DM2/M1 DREER DLDR									
DP/E ST CT LT									

6 Summary and future work

This paper advocates using dynamic changes in the term structure of interest rates to observe and assess the sign of financial and economic downturns, providing a computable framework for signalling and monitoring the financial crisis in an economy. The study first reviewed existing theories of the term structure of interest rate and the early warning indicators of financial crisis and identified key dimensions of the financial market for the purpose of financial stability and financial crisis prevention. The data of spot rates of Treasury Bonds in China's market from 2007 to 2019 were used to validate the Diebold-Li model in fitting the term structure of interest rates. By constructing proxy variables, it is confirmed that the factors L_t, S_t and C_t are suitable to represent the long-term interest rate, the difference between long and short-term interest rate and curvature respectively. Altogether, they effectively describe the shape of the term structure of interest rates. Five representative indicators, namely, CPI, loan-to-deposit ratio, M2/M1, real effective exchange rate and P/E ratio, were identified and incorporated in the construction of the VAR model for the design of an early warning system for financial crises.

By impulse response analysis and variance decomposition, we found that the early warning indicators of financial crises have the most compelling effect on the curvature factor C_t, followed by the slope factor S_t, whereas little effect was observed on the level factor L_t. Among these five indicators, DREER (Real Effective Exchange Rate) contributed most to the slope factor S_t and the curvature factor C_t and DM2/M1 has a good potential for L_t prediction. In sum, the term structure of Treasury bond interest rates plays a critical role in the early warning of a financial crisis. One can recognise and predict the signs of financial crises by analysing the change of the slope factor S_t and the curvature factor C_t of the term structure of interest rates. To be specific, the change of CPI and the effective exchange rate can be traced to make useful predictions and conclusions for the economic system, which is in line with much published research (Cushman, 1988; Gilchrist et al., 2017; Prowantaa and Ratnawati, 2018; Smallwood, 2019). The negative relation between the medium- or short-term bond rate and the DREER, which is our most predictive factor, is consistent with the Table 1 reported by Hsing (2015), where the regression coefficient between the government bond yield and exchange rate is negative. Ping (2004) demonstrated (Figure 3 of the paper) that the impulse response of the inflation rate with respective to M2 is positive in the Ireland market, since high inflation expectation has positive correlation with the long-term bond rate. The result also implicitly agrees with our finding in the impulse response test of the long-term rate with respect to the shock in M2/M1.

The idea of using bond yield dynamics as a barometer for financial markets or the economic outlook in general is not brand new. There have been abundant previous studies that either implicitly assumed such a premise in research (Demirgüç-Kunt and Detragiache, 1997; Zhuang and Dowling, 2002; Ercolani and Natoli, 2020; Samitas and Kenourgios, 2020; Suimon et al., 2020) or verified such observations with empirical data for various aspects of selected economies or markets (Yang, 2020; Bluwstein et al., 2020; Tillmann, 2020). In particular, it has been widely believed that the unusual movement of term structures, typically an inverted yield curve, may likely foretell a high probability of economic recession (Bauer and Mertens, 2018; Benzoni and Kelly, 2018; Quinn et al., 2021). One recent example is that the US treasury bond market had seen inverted curves in 2019, leading to a downturn in the economic outlook by economists and market practitioners alike (Yilmaz, 2019; Tokic, 2019; Gräb and Titzck, 2020), which is largely consistent with what had been observed several months later in the economy, including a series of stimulus initiatives from the US Fed (Feldkircher et al., 2021). Another major world market, the Japanese financial market, also demonstrated similar inverted yield curves in recent years, followed by observable signs of recessions.

Unlike "recession", the meaning of which is broader and the measurement of which has established indices such as the well-known NBER recession index, the term "financial crisis" is lacking a strict definition. In reality, a financial crisis is declared when various economic parameters, notably GDP growth rate, CPI, and employment rate, simultaneously and greatly deteriorate. As such, there have not been many historically recognised financial crises. The 1929 and 2008 financial crises are arguably two major ones that have elicited the most extensive research. Depending on one's perspective, there could be other incidents, such as the oil crisis and interest hike in late 1970s or the stock market crash in 1987, added to the list. Nevertheless, the total number of financial crises is definitely small from a standard econometric modelling point of view. In addition, economic growth may be influenced by unexpected rare events, possibly not considered by endogenous or exogenous models. For instance, the aforesaid countermeasures by the US Fed in response to the inverted bond yield curves in 2019 have likely been overtaken by economic effects of the pandemic in 2020. Thus the real effect of the Fed's response for improving the economy was not as measurable as in other economic cycles.

Given such challenges, the approach introduced in the current paper is not only novel but also a realistic choice to shed insights on the economy of the contemporary era. One limitation, though, is that there are many more economic indicators in reality to be included. But based on comparability and data availability, only five representative indicators among three categories (the macroeconomic indicators, the financial system indicators and risk of bubbles indicators) were selected by the current study. It could be the case that the selection of indicators is not yet comprehensive. Additionally, enhancement of technology in big data analysis and block-chain, for instance, may provide supplemental approaches to the early warning of financial crises, the study of which is a plausible future direction (Kou et al., 2019; Samitas et al., 2020).

References

Aslanidis, N. and Demiralp, S. (2020) 'Has the financial crisis affected the real interest rate dynamics in Europe?', *Journal of Business Cycle Research*, Vol. 16, No. 1, pp.1–18.

Bauer, M.D. and Mertens, T.M. (2018) 'Economic forecasts with the yield curve', *FRBSF Economic Letter*, Vol. 7, pp.8-07.

Baz, J., Sapra, S. and Ramirez, G. (2019) 'Stocks, bonds, and causality', *The Journal of Portfolio Management*, Vol. 45, No. 4, pp.37–48.

Bilson, F.O. (1979) 'Leading indicators of currency devaluations', *Columbia Journal of World Business*, Vol. 14, No. 4, pp.62–76.

Bluwstein, K., Buckmann, M., Joseph, A., Kang, M., Kapadia, S. and Simsek, Ö. (2020) *Credit Growth, The Yield Curve and Financial Crisis Prediction: Evidence from a Machine Learning Approach*. ECB Working Paper No. 2021/2614, Available at: http://dx.doi.org/10.2139/ssrn.3969562

Cai, Y., Chou, R.Y. and Li, D. (2009) 'Explaining international stock correlations with CPI fluctuations and market volatility', *Journal of Banking and Finance*, Vol. 33, No. 11, pp.2026–2035.

Chen, F.W., Feng, Y. and Wang, W. (2018) 'Impacts of financial inclusion on non-performing loans of commercial banks: evidence from China', *Sustainability*, Vol. 10, No. 9, p.3084.

Chen, S., Ma, H. and Mu, C. (2009) 'Research on MS-vAR model and regime state of financial risk early warning in China', *The Journal of Social Science*, Vol. 49, No. 1, pp.110–119.

Csepregi, D. (2020) *Residual Trading with WTI Futures Using the Four-Factor Diebold-Li Method*, Doctoral dissertation, BCE.

Culbertson, J.M. (1957) 'The term structure of interest rates', *Quarterly Journal of Economics*, Vol. 71, No. 4, pp.485–517.

Cushman, D.O. (1988) 'Exchange-rate uncertainty and foreign direct investment in the United States', *Weltwirtschaftliches Archiv*, Vol. 124, No. 2, pp.322–336.

Demirgüç-Kunt, A. and Detragiache, E. (1997) *The Determinants of Banking Crises-Evidence From Developing and Developed Countries* (*Vol.* 106) World Bank Publications.

Diebold, F.X. and Li, C. (2002) 'Forecasting the term structure of government bond yields', *SSRN Electronic Journal*, Vol. 130, No. 2, pp.337–364, doi: 10.1016/j. jeconom.2005.03.005.

Diebold, F.X., Rudebusch, G.D. and Boragan, A.S. (2006) 'The macroeconomy and the yield curve: a dynamic latent factor approach', *The Journal of Econometrics*, Vol. 131, Nos. 1–2, pp.309–338.

Ercolani, V. and Natoli, F. (2020) 'Forecasting US recessions: the role of economic uncertainty', *Economics Letters*, Vol. 193, p.109302.

Fama, E.F. (1984) 'Forward and spot exchange rates', *The Journal of Monetary Economics*, Vol. 14, No. 3, pp.319–338, doi: 10.1016/0304-3932(84)90046-1.

Fama, E.F. and Bliss, R.R. (1987) 'The information in long-maturity forward rates', *American Economic Review*, Vol. 77, No. 4, pp.680–692.

Feldkircher, M., Huber, F. and Pfarrhofer, M. (2021) 'Measuring the effectiveness of US monetary policy during the COVID-19 recession', *Scottish Journal of Political Economy*, doi: 10.1111/sjpe.12275, Advance Online Publication. https://doi.org/10.1111/sjpe.12275

Gilchrist, S., Schoenle, R., Sim, J. and Zakrajšek, E. (2017) 'Inflation dynamics during the financial crisis', *American Economic Review*, Vol. 107, No. 3, pp.785–823.

Gräb, J. and Titzck, S. (2020) 'US yield curve inversion and financial market signals of recession', *Economic Bulletin Boxes*, p.1.

Han, C., Wang, Y. and Xu, Y. (2019) 'Efficiency and multifractality analysis of the Chinese stock market: evidence from stock indices before and after the 2015 stock market crash', *Sustainability*, Vol. 11, No. 6, p.1699.

Hicks, J.E. (1939) *Value and Capital*, 2nd ed., 1946, Clarendon Press, Oxford.

Horne, V.J. (1965) *Liquidity Preference*, interest-rate risk, and the term structure of interest rates', *The Journal of Finance*, Vol. 20, No. 3, pp.515–516, doi: 10.2307/2978009.

Klopotan, I., Zoroja, J. and Meško, M. (2018) 'Early warning system in business, finance, and economics: bibliometric and topic analysis', *International Journal of Engineering Business Management*, Vol. 10, p.1847979018797013.

Kou, G., Chao, X., Peng, Y., Alsaadi, F.E. and Herrera-Viedma, E. (2019) 'Machine learning methods for systemic risk analysis in financial sectors', *Technological and Economic Development of Economy*, Vol. 25, No. 5, pp.716–742.

Langetieg, T.C. and Smoot, S.J. (1989) 'Estimation of the term structure of interest rates', *The Journal of Financial Services Research*, No. 1, pp.181–222.

Li, P., Jiang, F. and Cao, J. (2019) *Industrial Overcapacity and Duplicate Construction in China: Reasons and Solutions (Vol. 15)*, World Scientific, NJ.

Longstaff, F.A. (2000) 'Arbitrage and the expectations hypothesis', *The Journal of Finance*, Vol. 55, No. 2, pp.989–994, doi: 10.1111/0022-1082.00234.

Lütkepohl, H. (2005) *New Introduction to Multiple Time Series Analysis*, Springer Science and Busniess Media, Heidelberg, pp.186–189, doi:10.1007/978-3-540-27752-1.

McCulloch, J.H. (1971) 'Measuring the term structure of interest rates', *The Journal of Business*, Vol. 44, No. 1, pp.19–31.

Modigliani, F. and Sutch, R. (1966) 'Innovations in interest rate policy', *American Economic Review*, Vol. 56, No. 2, pp.178–197.

Nelson, C.R. and Siegel, A.F. (1987) 'Parsimonious modelling of yield curves', *The Journal of Business*, Vol. 60, No. 4, pp.473–489, doi: 10.1086/296409.

Poghosyan, K. and Poghosyan, A. (2019) 'Yield curve estimation and forecasting in Armenia', *Armenian Journal of Economics*, Vol. 4, pp.1–21.

Prowantaa, E. and Ratnawati, K. (2018) 'The effect of GDP, inflation, exchange rate, interest rate on stock price index in early warning system: an empirical study in ASEAN-5 countries', *Archives of Business Research (ABR)*, Vol 6, No. 12, pp.90–100.

Reinhart, C.M. and Rogoff, K.S. (2008) 'Is the 2007 U.S. sub-prime financial crisis so different? An international historical comparison', *The American Economic Review*, Vol. 98, No. 2, pp.339–344, doi: 10.1257/aer.98.2.339.

Samitas, A., Kampouris, E. and Kenourgios, D. (2020) 'Machine learning as an early warning system to predict financial crisis', *International Review of Financial Analysis*, Vol. 71, p.101507.

Sari, M., Putri, S.R. and Wahyuningsih, N. (2019) 'Modelling of short rate (β2) parameter diebold-li model using Vasicek stochastic differential equations', *Journal of Physics: Conference Series*, Vol. 1218, No. 1, May, p.012059, IOP Publishing.

Shea, G.S. (1984) 'Pitfalls in smoothing interest rate term structure data: equilibrium model and spline estimations', *The Journal of Financial and Quantitative Analysis*, Vol. 19, No. 3, pp.253–269, doi: 10.2307/2331089.

Shi, M., Wang, S., Xu, S. and Tao, S. (2005) 'Empirical analysis on term structure of China interbank offered rates', *The Journal of Management Sciences in China*, Vol. 2005, No. 5, pp.43–49.

Smallwood, A.D. (2019) 'Analyzing exchange rate uncertainty and bilateral export growth in China: a multivariate GARCH-based approach', *Economic Modelling*, Vol. 82, pp.332–344.

Steeley, J.M. (1991) 'Estimating the gilt-edged term structure: basis splines and confidence intervals', *Journal of Business Finance and Accounting*, Vol. 18, No. 4, pp.13–29, doi: 10.1111/j.1468-5957.1991.tb01119.x.

Suimon, Y., Sakaji, H., Izumi, K. and Matsushima, H. (2020) 'Autoencoder-based three-factor model for the yield curve of Japanese government bonds and a trading strategy', *Journal of Risk and Financial Management*, Vol. 13, No. 4, p.82.

Taylor, M.M.P. (1991) *Modelling the Yield Curve*, International Monetary Fund.

Tillmann, P. (2020) 'Monetary policy uncertainty and the response of the yield curve to policy shocks', *Journal of Money, Credit and Banking*, Vol. 52, No. 4, pp.803–833.

Tokic, D. (2019) *Is Yield Curve Predicting a US Recession in 2020*?

Wang, P., Zong, L. and Ma, Y. (2020) 'An integrated early warning system for stock market turbulence', *Expert Systems with Applications*, Vol. 153, p.113463.

Wang, X. and Wang, H. (2010) 'Empirical test of the term structure of Shibor based on the expectation theory', *Operations Research and Management Science*, Vol. 19, No. 3, pp.124–130.

Wu, D. and Xie, C. (2005) 'Test of the expectation theory of the term structure of treasury market among China banks', *The Chinese Journal of Management*, Vol. 2005, No. 6, pp.536–539.

Yang, P.R. (2020) 'Using the yield curve to forecast economic growth', *Journal of Forecasting*, Vol. 39, No. 7, pp.1057–1080.

Appendix

Table A1 The vector autoregression estimates

	LT	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
LT(–1)	1.239197	0.077354	0.839092	0.212978	0.616159	–0.004942	–0.287350	1.099706
	(0.08735)	(0.19155)	(0.38795)	(0.28683)	(0.38105)	(0.03967)	(0.63742)	(1.85653)
	[14.18621]	[0.40384]	[2.16289]	[0.74252]	[1.61700]	[–0.12457]	[–0.45080]	[0.59235]
LT(–2)	–0.325357	–0.035071	–0.642649	–0.132906	–0.645353	0.031389	0.873750	–1.372032
	(0.09060)	(0.19868)	(0.40238)	(0.29750)	(0.39523)	(0.04114)	(0.66114)	(1.92561)
	[–3.59104]	[–0.17653]	[–1.59710]	[–0.44674]	[–1.63286]	[0.76289]	[1.32158]	[–0.71252]
ST(–1)	–0.013483	1.014051	0.475317	0.202808	0.240369	–0.005932	0.231269	–0.436852
	(0.04188)	(0.09183)	(0.18599)	(0.13751)	(0.18268)	(0.01902)	(0.30558)	(0.89004)
	[–0.32197]	[11.0427]	[2.55566]	[1.47487]	[1.31580]	[–0.31192]	[0.75681]	[–0.49083]
ST(–2)	0.019860	–0.145731	–0.199614	–0.259181	–0.209678	0.028389	–0.090452	0.673701
	(0.04303)	(0.09437)	(0.19112)	(0.14131)	(0.18772)	(0.01954)	(0.31403)	(0.91462)
	[0.46150]	[–1.54431]	[–1.04442]	[–1.83416]	[–1.11694]	[1.45266]	[–0.28804]	[0.73659]
CT(–1)	0.031660	0.094564	0.643465	0.283660	–0.010160	–0.007096	–0.054882	–0.554831
	(0.02068)	(0.04536)	(0.09186)	(0.06792)	(0.09023)	(0.00939)	(0.15093)	(0.43959)
	[1.53069]	[2.08496]	[7.00489]	[4.17662]	[–0.11260]	[–0.75544]	[–0.36362]	[–1.26215]

Table A1 The vector autoregression estimates (continued)

	LT	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
CT(–2)	–0.036137	–0.014580	0.020458	–0.194426	0.037719	–0.003472	0.286829	–0.000798
	(0.02015)	(0.04418)	(0.08949)	(0.06616)	(0.08790)	(0.00915)	(0.14703)	(0.42825)
	[–1.79341]	[–0.32998]	[0.22861]	[–2.93856]	[0.42912]	[–0.37943]	[1.95075]	[–0.00186]
DCPI(–1)	–0.023710	–0.003133	0.183897	0.333232	–0.110610	–0.003053	–0.490362	0.266121
	(0.02480)	(0.05439)	(0.11016)	(0.08145)	(0.10820)	(0.01126)	(0.18100)	(0.52718)
	[–0.95587]	[–0.05760]	[1.66933]	[4.09132]	[–1.02225]	[–0.27104]	[–2.70915]	[0.50480]
DCPI(–2)	0.019124	–0.020042	0.064642	0.005876	–0.104500	0.003772	–0.035754	–0.549856
	(0.02402)	(0.05267)	(0.10667)	(0.07887)	(0.10477)	(0.01091)	(0.17527)	(0.51048)
	[0.79620]	[–0.38054]	[0.60599]	[0.07450]	[–0.99739]	[0.34586]	[–0.20400]	[–1.07714]
DLDR(–1)	0.005100	–0.029356	0.077297	–0.123167	–0.277206	–0.003827	0.300677	–0.320915
	(0.01884)	(0.04130)	(0.08366)	(0.06185)	(0.08217)	(0.00855)	(0.13745)	(0.40033)
	[0.27073]	[–0.71072]	[0.92399]	[–1.99136]	[–3.37367]	[–0.44738]	[2.18754]	[–0.80162]
DLDR(–2)	–0.016822	–0.011470	–0.135763	–0.028708	–0.247698	–0.006129	–0.273784	0.551724
	(0.01851)	(0.04058)	(0.08219)	(0.06076)	(0.08073)	(0.00840)	(0.13504)	(0.39330)
	[–0.90902]	[–0.28265]	[–1.65189]	[–0.47245]	[–3.06841]	[–0.72933]	[–202747]	[1.40279]
DM2/M1(–1)	–0.022951	–0.592929	0.417190	–0.331712	–1.994877	–0.146180	–2.313501	0.936968
	(0.19524)	(0.42813)	(0.86710)	(0.64109)	(0.85168)	(0.08866)	(1.42469)	(4.14951)
	[–0.11755]	[–1.38494]	[0.48113]	[–0.51742]	[–2.34228]	[–1.64871]	[–1.62386]	[0.22580]
DM2/M1(–2)	0.306105	–0.268602	0.721544	–0.023716	–1.351949	–0.067257	2.222710	–2.418091
	(0.19703)	(0.43205)	(0.87505)	(0.64697)	(0.85949)	(0.08948)	(1.43776)	(4.18757)
	[1.55358]	[–0.62169]	[0.82457]	[–0.03666]	[–1.57296]	[–0.75167]	[1.54595]	[–0.57744]
DREER(–1)	–0.008600	0.027016	–0.108302	0.124915	0.010700	0.002493	0.442485	–0.166794
	(0.01160)	(0.02544)	(0.05153)	(0.03810)	(0.05061)	(0.00527)	(0.08467)	(0.24660)
	[–0.74119]	[1.06183]	[–2.10168]	[3.27864]	[0.21141]	[0.47317]	[5.22611]	[–0.67637]
DREER(–2)	0.009922	–0.056487	–0.115205	–0.111530	–0.018587	0.001208	–0.149879	0.083827
	(0.01219)	(0.02673)	(0.05413)	(0.04002)	(0.05317)	(0.00554)	(0.08894)	(0.25905)
	[0.81403]	[–2.11347]	[–2.12825]	[–2.78670]	[–0.34957]	[0.21825]	[–1.68515]	[0.32360]
DP/E(–1)	–0.002012	0.005878	–0.005903	0.017132	0.035166	–0.002002	–0.025743	–0.084562
	(0.00390)	(0.00855)	(0.01731)	(0.01280)	(0.01701)	(0.00177)	(0.02845)	(0.08285)
	[–0.51602]	[0.68764]	[–0.34093]	[1.33834]	[2.06791]	[–1.13069]	[–0.90497]	[–1.02063]
DP/E(–2)	0.007786	–0.000756	–0.002529	–0.006871	–0.017578	–7.78E–05	0.031129	0.176475
	(0.00398)	(0.00872)	(0.01766)	(0.01306)	(0.01735)	(0.00181)	(0.02902)	(0.08453)
	[1.95755]	[–0.08666]	[–0.14317]	[–0.52616]	[–1.01319]	[–0.04307]	[1.07261]	[2.08777]
C	0.359774	–0.251802	–0.856038	–0.275052	0.343013	–0.085589	–1.737043	0.474114
	(0.12835)	(0.28144)	(0.57001)	(0.42144)	(0.55987)	(0.05828)	(0.93655)	(272778)
	[2.80317]	[–0.89470]	[–1.50180]	[–0.65265]	[0.61266]	[–1.46847]	[–1.85472]	[0.17381]

Table A1 The vector autoregression estimates (continued)

	LT	*ST*	*CT*	*DCPI*	*DLDR*	*DM2/M1*	*DREER*	*DP/E*
R-squared	0.887952	0.843516	0.672894	0.349815	0.260832	0.101698	0.358316	0.124441
Adi. R-squared	0.874770	0.825106	0.634410	0.273322	0.173871	–0.003985	0.282824	0.021434
Sum sq. resids	2.855920	13.73255	56.33095	30.79261	54.34522	0.588969	152 0720	1290.034
S.E. equation	0.144912	0.317765	0.643582	0.475832	0.632137	0.065808	1.057439	3.079861
F-statistic	67.36028	45.81871	17.48543	4.573194	2.999414	0.962295	4.746395	1.208080
Loq likelihood	87.45225	–32.68141	–140.6593	–94.45562	–137.9139	208.2286	–216.6322	–380.1945
Akaike AIC	–0.920944	0.649430	2.060905	1.456936	2.025018	–2.49972	3.054016	5.192085
Schwarz SC	–0.584229	0.986146	2.397621	1.793651	2.361733	–2.163005	3.390731	5.528800
Mean dependent	4.157966	–1.615521	–1.553796	0.001307	0.072876	0.003922	0.158301	–0.194837
S.D. dependent	0.409495	0.759835	1.064406	0.558191	0.695484	0.065677	1.248654	3.113408
Determinant resid covariance (dof adj.)	1.95E-06							
Determinant resid covariance	7.59E-07							
Log likelihood	–658.7554							
Akaike information criterion	10.38896							
Schwarz criterion	13.08268							
Number of coefficients	136							