

Traveling Waves

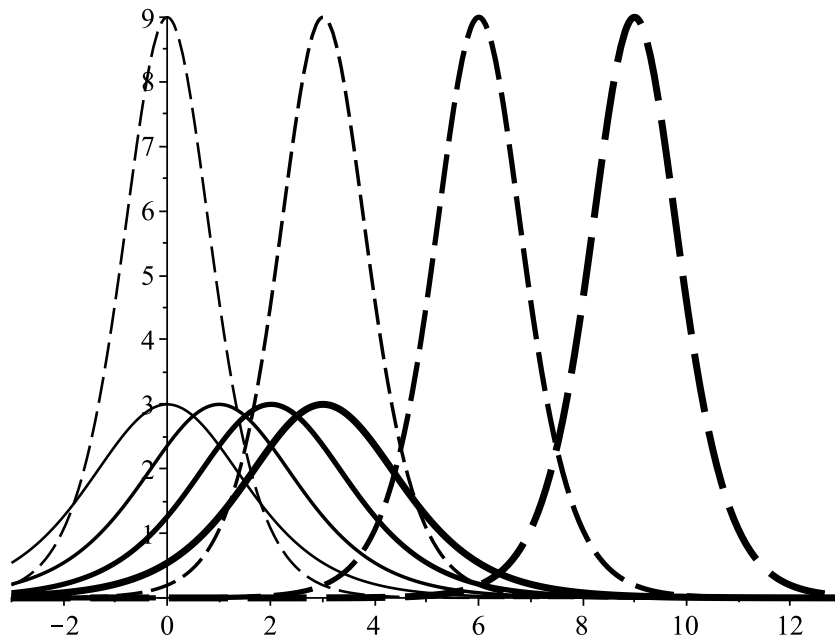
Consider the Korteweg-de Vries equation

$$\frac{\partial \eta}{\partial t} + \eta \frac{\partial \eta}{\partial x} + \frac{\partial^3 \eta}{\partial x^3} = 0, \quad -\infty < x < \infty, \quad \eta(x, t) \rightarrow 0 \text{ as } |x| \rightarrow \infty.$$

In class we found a traveling-wave solution to be

$$\eta(x, t) = 3V \operatorname{sech}^2 \left(\frac{\sqrt{V}(x - Vt)}{2} \right). \quad (1)$$

Graphs of the solution are shown below. Note that waves with larger amplitude are narrower and travel faster.



Graph of (1) for $V = 1$ (solid) and $V = 3$ (dashed).
In increasing order of thickness: $t = 0, 1, 2, 3$.

