

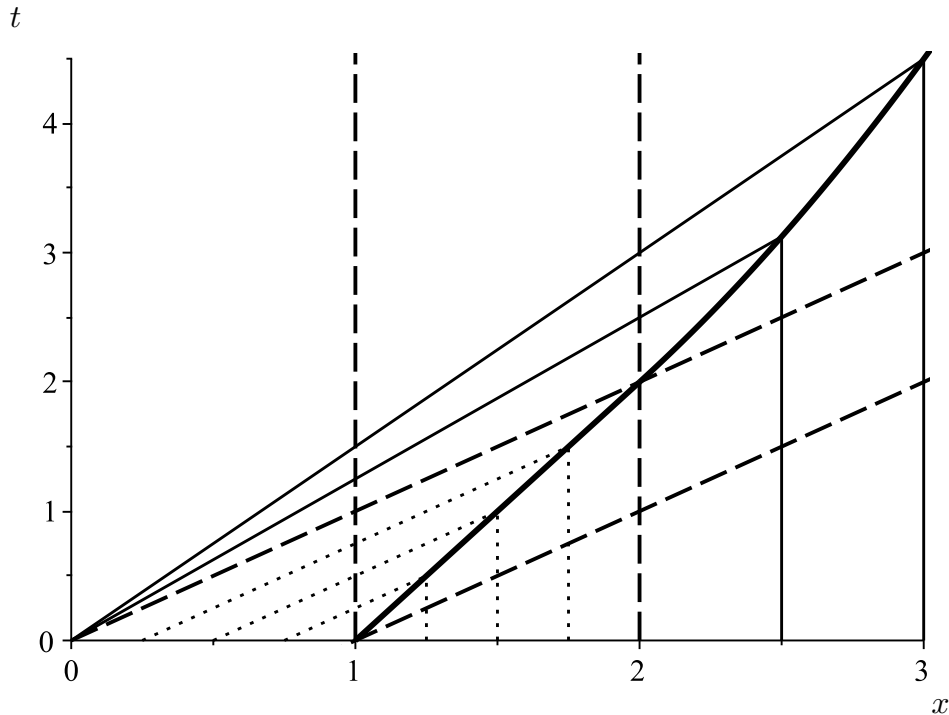
# Shock Structure

For the problem

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad u(x, 0) = \begin{cases} 1, & x \in [0, 1], \\ 0, & \text{else,} \end{cases} \quad (1)$$

we determined that the shock position is given by

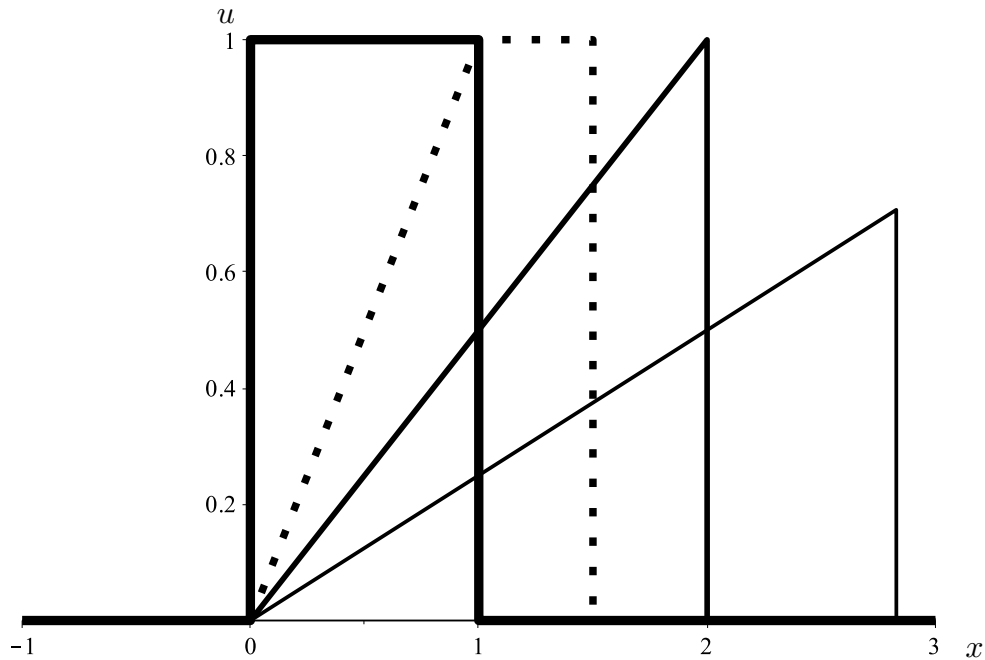
$$s(t) = \begin{cases} 1 + t/2, & t \in [0, 2], \\ \sqrt{2t}, & t \geq 2. \end{cases} \quad (2)$$



Dotted lines: characteristics that intersect at the shock for  $t \in [0, 2]$  ( $u = 1$  and  $u = 0$ ). Dashed lines intersecting at  $(1, 0)$ : envelope of crossings. Thin solid lines: characteristics that intersect at the shock for  $t \geq 2$  (expansion wave and  $u = 0$ ). Dashed lines intersecting at  $(2, 2)$ : first crossing with expansion wave. Thick solid curve: front position.

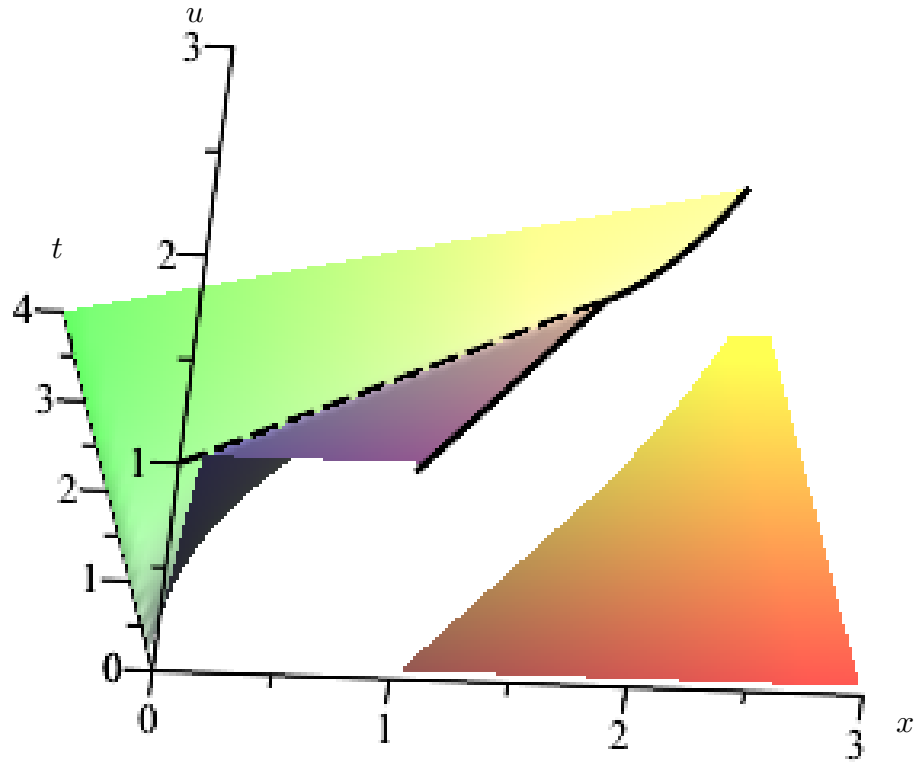
The characteristic diagram is plotted above. Note that the envelope of crossings is given by the lines  $x = t + 1$  and  $x = 1$ . Note that the shock position changes from a straight line for  $x \in [1, 2]$  to a parabola at  $(2, 2)$ .

# Shock Profiles



Profiles for  $t = 0, 1$  (dotted line), 2, and 4 (in decreasing order of thickness).

Profiles for  $u$  are shown above. Note that for  $t = 4$  (once the fan characteristics determine the front position), the overall height of the sawtooth declines.



$u$  vs.  $x$  and  $t$ . Solid line is shock position; dotted line is extent of expansion wave.

