

Quasilinear Equations

For the problem

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial \rho}{\partial x} = 0, \quad \rho(x, 0) = \tanh x,$$

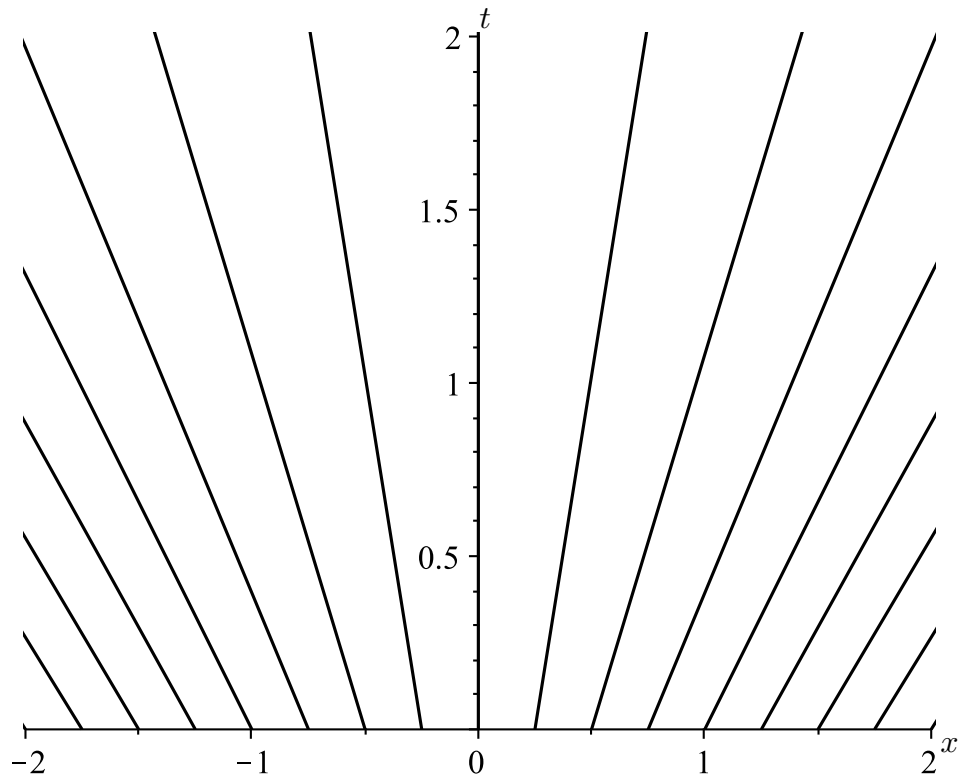
we determined that the solution is given by

$$\rho = \tanh \xi \tag{1a}$$

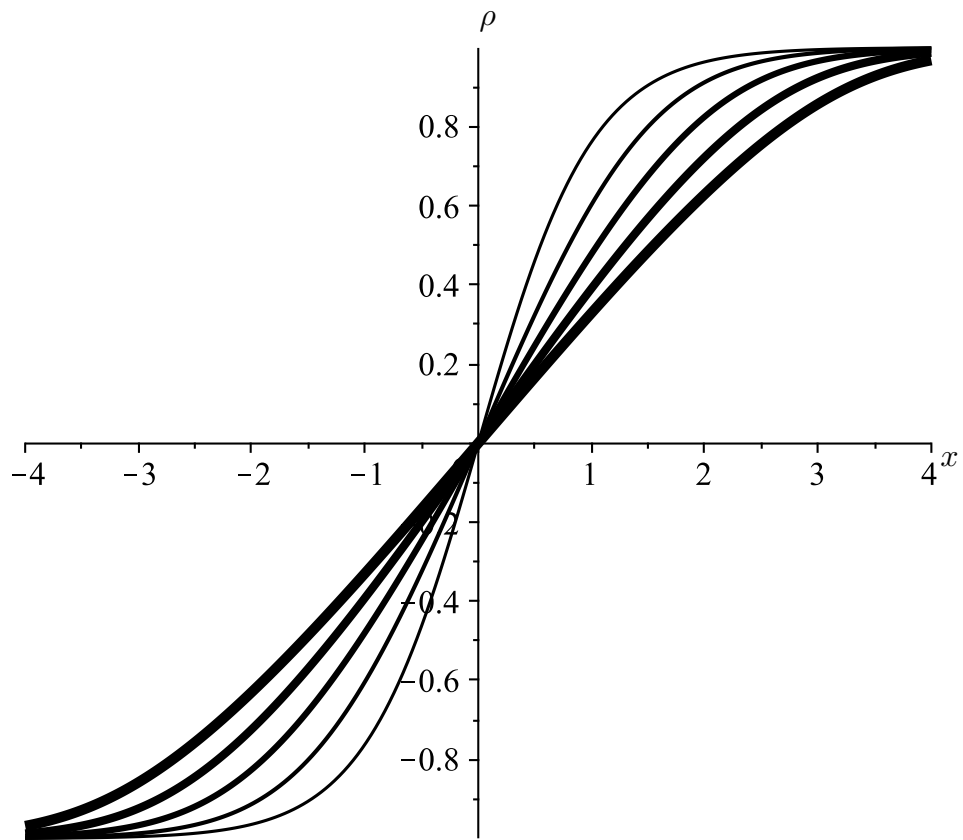
along characteristics given by

$$x = \xi + (\tanh \xi)t. \tag{1b}$$

Below are plotted characteristics given by (1) for $\xi = -0.8$ through $\xi = 0.8$ incremented by 0.1. Note the expansion fan of the characteristics.



Characteristics given by (1b).



$\rho(x, t)$ vs. x as given by (1) with $t = 0, 0.5, 1, 1.5, 2$ (in increasing order of thickness). Since the wave speed is the same as the height, the higher parts pull away from the center, flattening the curve.

