# The Navier-Stokes Equations

In Cartesian coordinates, the two-dimensional steady Navier-Stokes equations (neglecting gravity) are given by conservation of mass:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \tag{1m}$$

conservation of $\tilde{x}$-momentum:

$$\rho\left(\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right) = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \mu\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right), \tag{1$x$}$$

and conservation of $\tilde{y}$-momentum:

$$\rho\left(\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right) = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \mu\left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right). \tag{1$y$}$$

## Unidirectional Flow

In the case of unidirectional flow in the $\tilde{x}$-direction, these may be reduced to the following equations:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} = 0, \tag{2m}$$

$$0 = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \mu\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}, \tag{2$x$}$$

$$0 = -\frac{\partial \tilde{p}}{\partial \tilde{y}}. \tag{2$y$}$$