First-Order Linear Equations (Revised)

For the problem

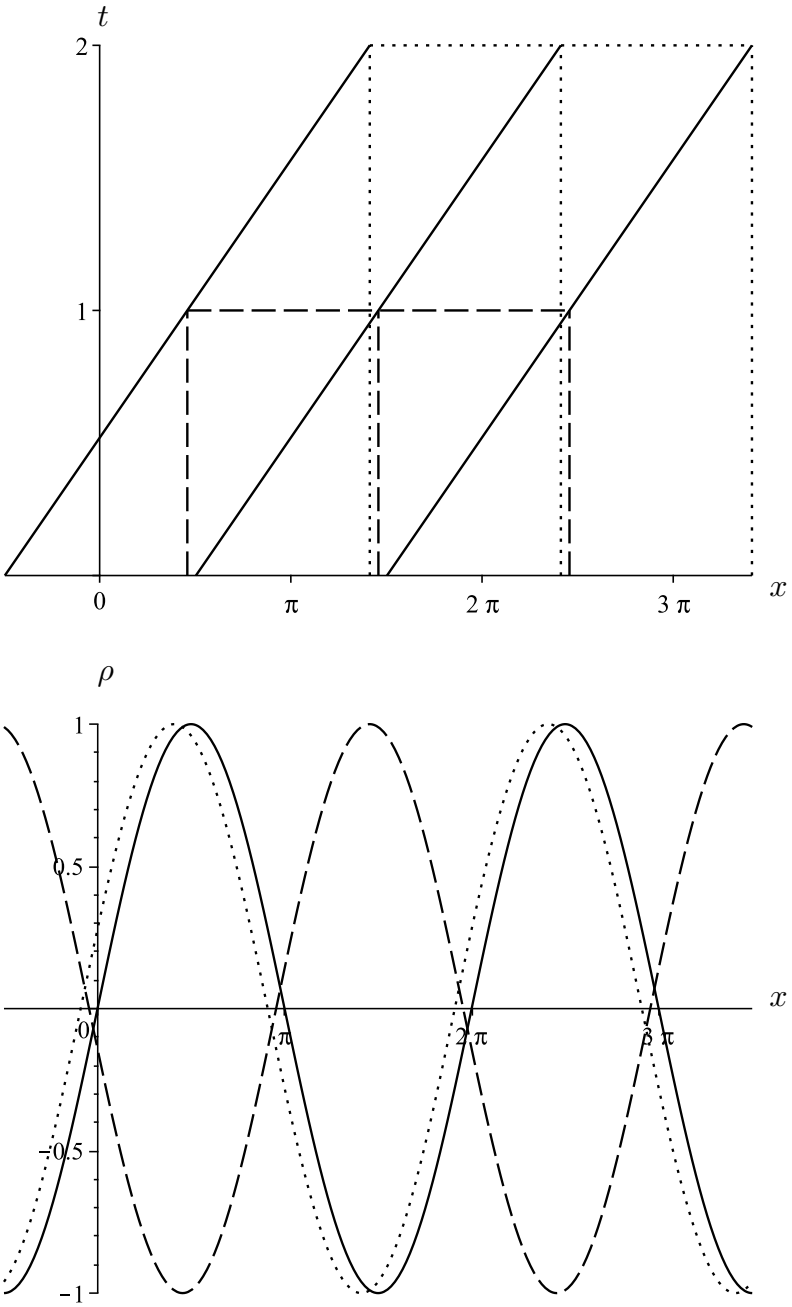
$$\frac{\partial \rho}{\partial t} + 3\frac{\partial \rho}{\partial x} = 0, \qquad \rho(x,0) = \sin x,$$

we determined that the solution is given by

$$\rho(x,t) = \sin(x - 3t),$$

which is just a traveling wave.

The characteristics and solution are shown on the next page.



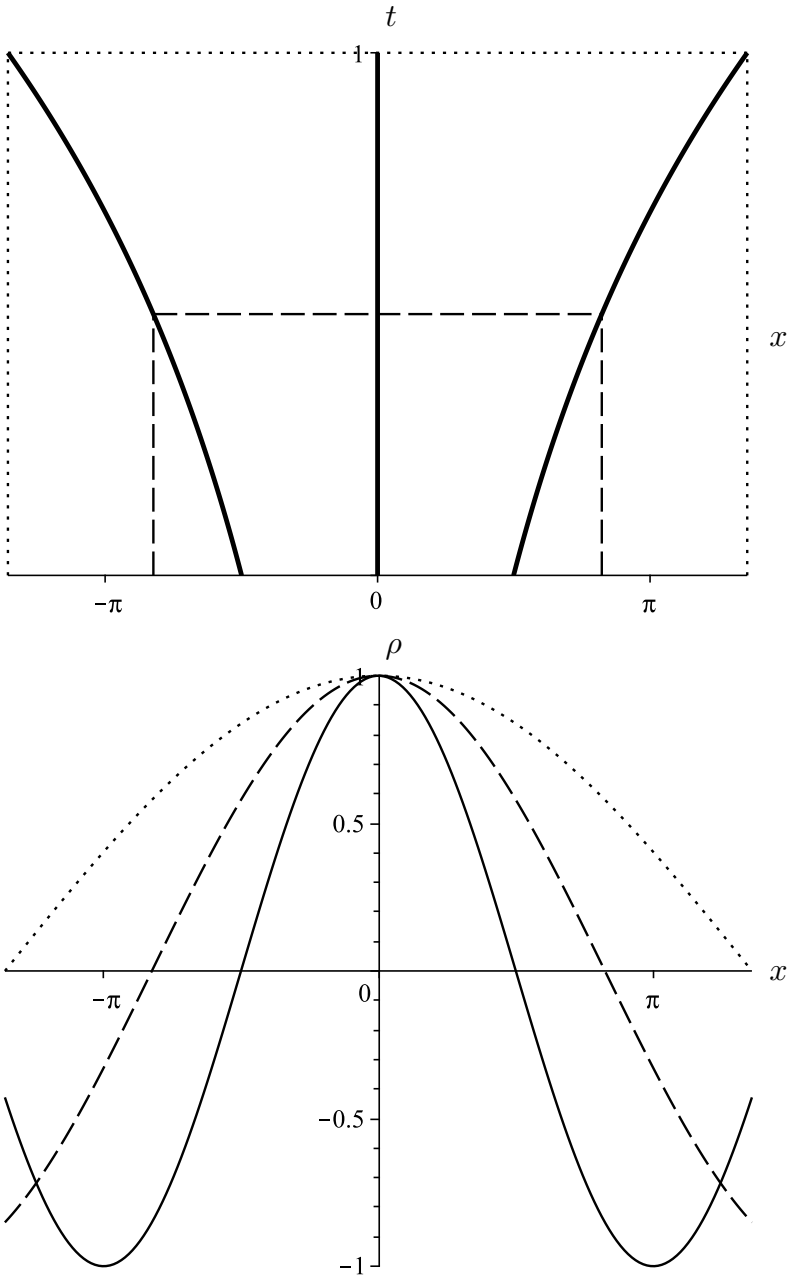
The top graph shows characteristics in the x-t plane. The solid lines correspond to the characteristics for the first three extrema. The dashed lines indicate where those points have shifted to after $t = 1$, and the dotted lines show where those points have shifted to after $t = 2$. The bottom graph shows the solution for $t = 0$ (solid curve), $t = 1$ (dashed curve), and $t = 2$ (dotted curve).

For the problem

$$\frac{\partial \rho}{\partial t} + x\frac{\partial \rho}{\partial x} = 0, \qquad \rho(x,0) = \cos x,$$

we determined that the solution is given by

$$\rho(x,t) = \cos(xe^{-t}).$$



The graphs are similar to those before, except the lines correspond to the characteristics for the maximum and the surrounding zeroes. Also, the snapshots are taken at $t = 0$, $1/2$, and 1. Note that the wave spreads out over time.