MATH 616-010 Prof. D. A. Edwards Modeling in Applied Mathematics Sept. 27, 2023

Liapunov Functions

For the case where

$$\begin{aligned} \dot{x} &= -y - x^3 \\ \dot{y} &= x - y^3 \end{aligned} \tag{1}$$

we found that a Liapunov function was given by

$$V = x^2 + y^2,$$

and that the origin is stable.



Phase plane of (1). Note the slow approach to the origin.

For the case where

$$\begin{aligned} \dot{x} &= -y + 3x^3\\ \dot{y} &= 2x + y^3 \end{aligned} \tag{2}$$

we found that a Liapunov function was given by

$$V = 2x^2 + y^2,$$

and that the origin is unstable.



For the case where

$$\dot{x} = -y - x(x - 1) \dot{y} = (2x - 1)[x(x - 1) - y]$$
(3)

we found that a Liapunov function was given by

$$V = x^2(x-1)^2 + y^2.$$

Here the origin is unstable and (1,0) is stable.



