Epidemics (Revised)

Consider the SIR model for an epidemic:

$$\begin{aligned}\dot{S} &= -\frac{SI}{\rho}, \\ \dot{I} &= \left(\frac{S}{\rho} - 1\right) I, \\ \dot{R} &= I.\end{aligned} \tag{1}$$

subject to

$$S(0) = S_0, \qquad I(0) = 1 - S_0, \qquad R(0) = 0.$$

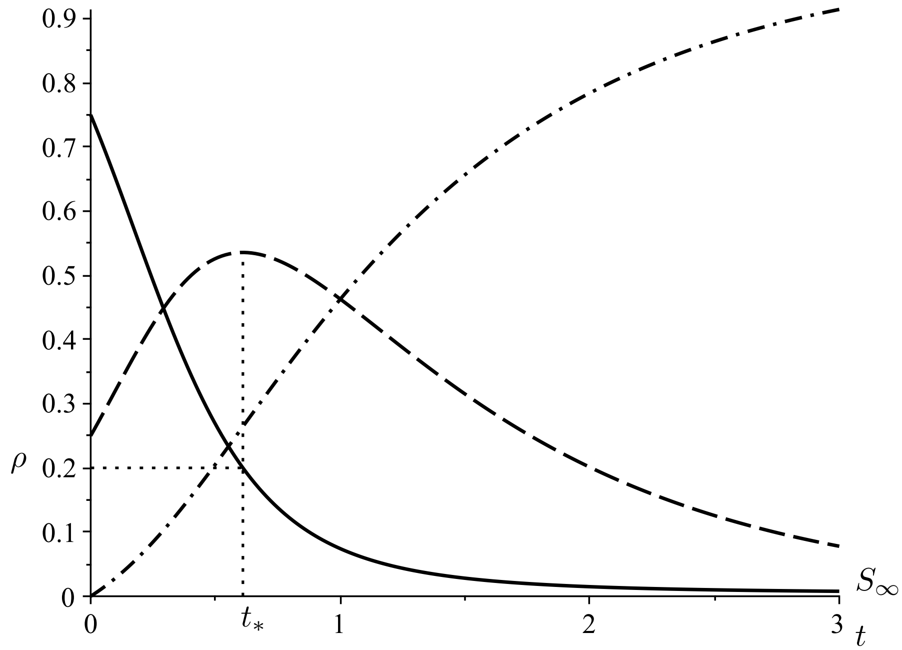
By rewriting I in terms of S, we found that

$$I(S) = 1 - S - \rho \log\left(\frac{S_0}{S}\right), \tag{2}$$

where the log is always positive since S is always decreasing. We also were able to compute that S_∞, the steady state of S, satisfied

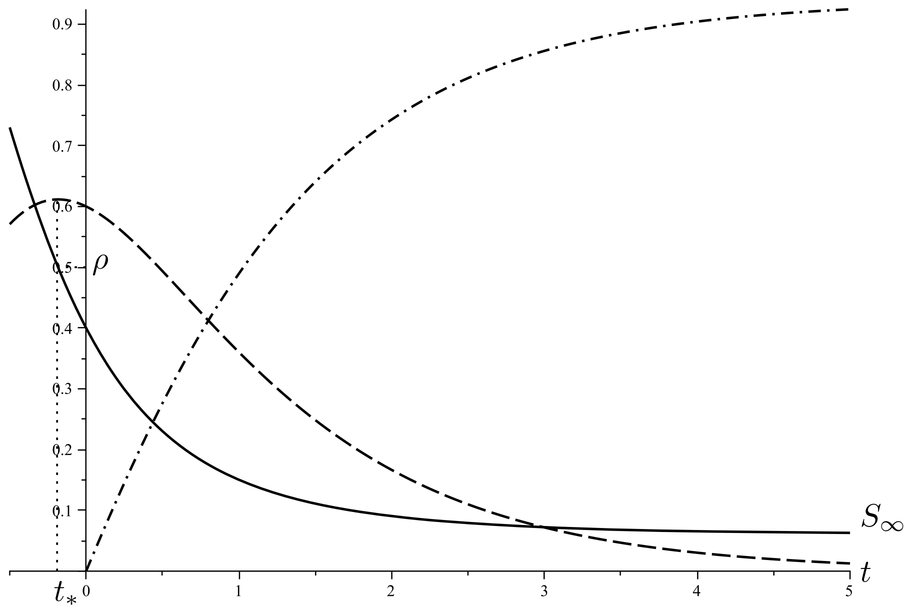
$$S_\infty = S_0 \exp\left(-\frac{1 - S_\infty}{\rho}\right). \tag{3}$$

We determined that if $S_0 < \rho$, there is an epidemic until some time t_* defined by $S(t_*) = \rho$, where I will reach a maximum. That case is illustrated below.



Results from (1) with $S_0 = 0.75$, $\rho = 0.2$. Solid curve: $S(t)$. Dashed curve: $I(t)$. Dash-dot curve: $R(t)$.

If $S_0 < \rho$, there is no epidemic and the maximum value of I occurs at some (unphsy-ical) time $t_* < 0$. That case is illustrated below.



Results from (1) with $S_0 = 0.4$, $\rho = 0.5$. Solid curve: $S(t)$. Dashed curve: $I(t)$. Dash-dot curve: $R(t)$.