MATH 616-010 Modeling in Applied Mathematics
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Conservative Systems

We also examined the case where

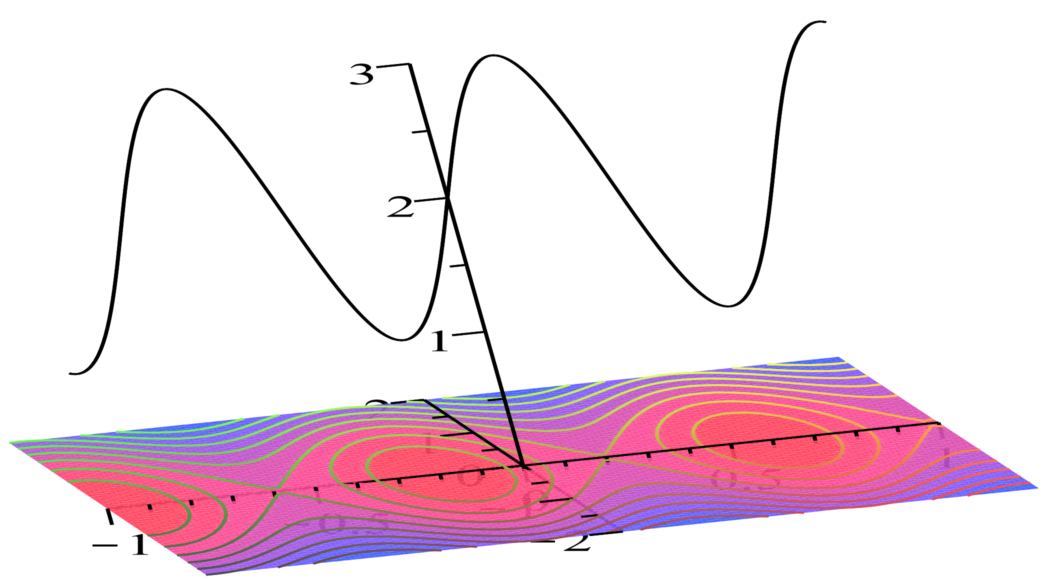
$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -V'(x) \end{aligned} \qquad \Longrightarrow \qquad H = V(x) + \frac{y^2}{2} + C.$$

In this case, the *conservative* case, H has a physical interpretation as an energy. We also noted that the stability of the fixed points (which occur at critical points of V) may be surmised by examining the curvature at the critical points.

We consider the case where

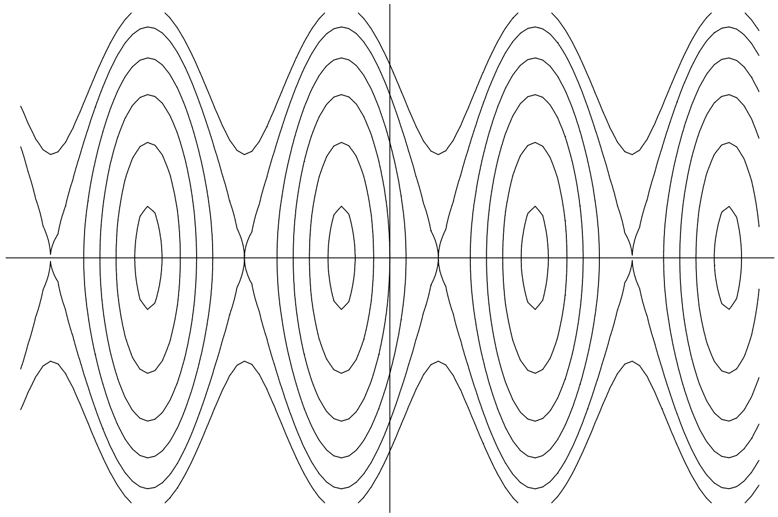
$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -8\cos 8x \end{aligned} \qquad \Longrightarrow \qquad H = \sin 8x + \frac{y^2}{2} + C.$$

Here $V(x) = \sin 8x$, so we would expect an infinite series of alternating centers and saddles.



$V(x) = \sin 8x + 2$ with trajectories of conservative system.

This figure shows the potential function as well as the trajectories. Note the saddles occur at maxima of $V(x)$, while centers occur at minima of $V(x)$.



Trajectories of conservative system.

We isolate the trajectories in the above diagram. Note that the saddles are connected by *heteroclinic orbits*.