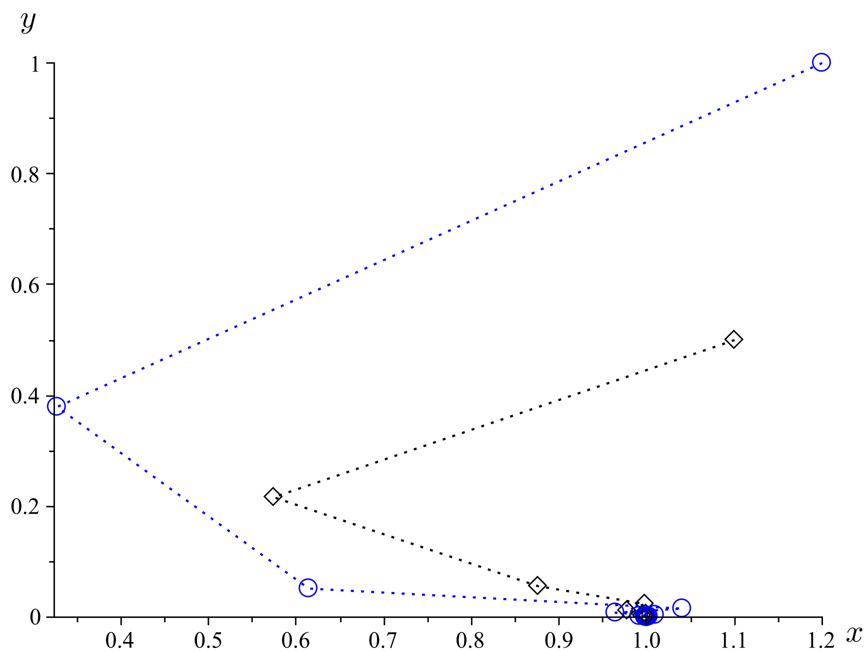
Beddington Model (Revised)

Consider the Beddington model presented in class:

$$x_{t+1} = x_t \exp\left(r(1-x_t) - y_t\right), \tag{3a}$$

$$y_{t+1} = c(1-e^{-y_t})x_t, \qquad c = \beta a K. \tag{3b}$$

The fixed point at the origin is unstable to perturbations in the host. The fixed point at $(1, 0)$ is stable for $r < 2$, $c < 1$, as shown below.



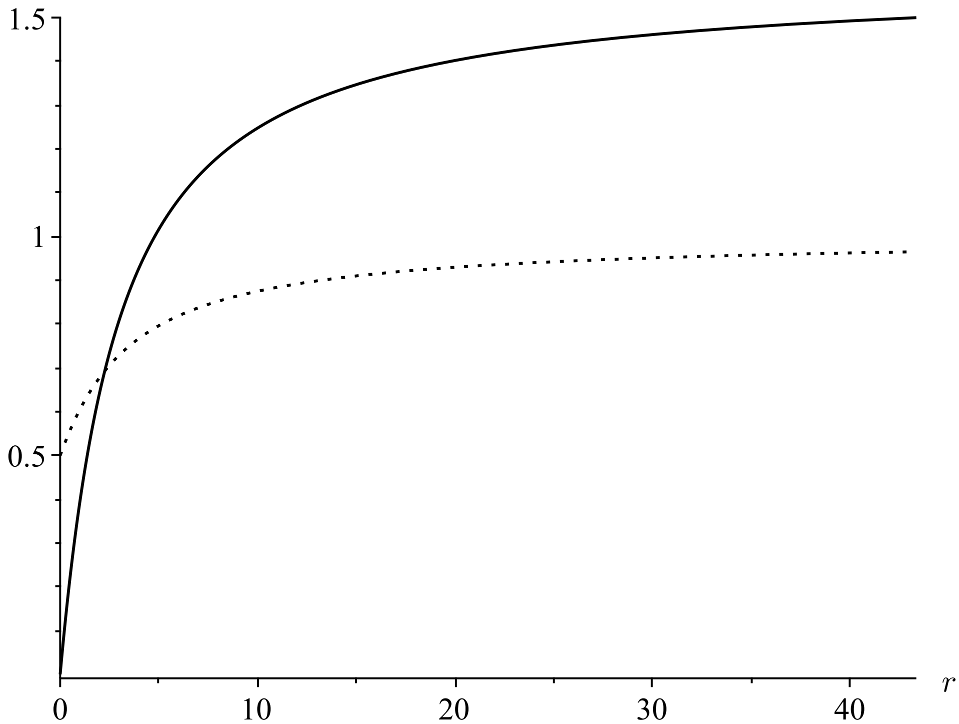
Iterates of (3) for $r = 3/2$, $c = 1/2$. Fixed point at $(1, 0)$ is stable. (Animation online.)

Since $c < 1$, the parasitoids do not produce enough offspring to keep their population from going to 0. Since $r < 2$, the hosts do not produce enough offspring to produce oscillations in their population.

If $c > 1$, the parasitoids reproduce at a high enough rate that there is a third fixed point with nonzero values for both populations given by

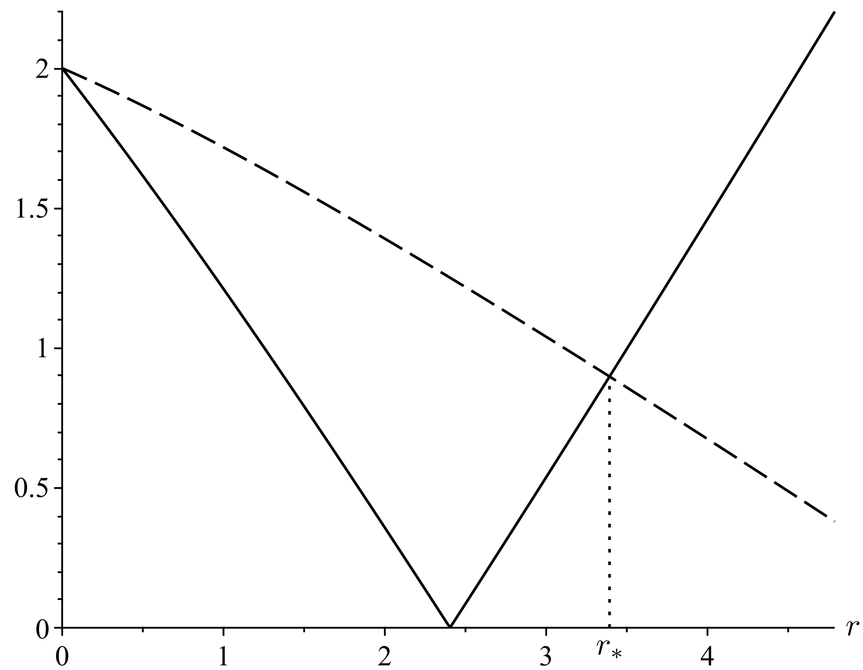
$$x_* = \frac{y_*}{c(1 - e^{-y_*})}, \qquad r = y_* \left[1 - \frac{y_*}{c(1 - e^{-y_*})}\right]^{-1},$$

the coordinates of which are graphed below *vs.* r.



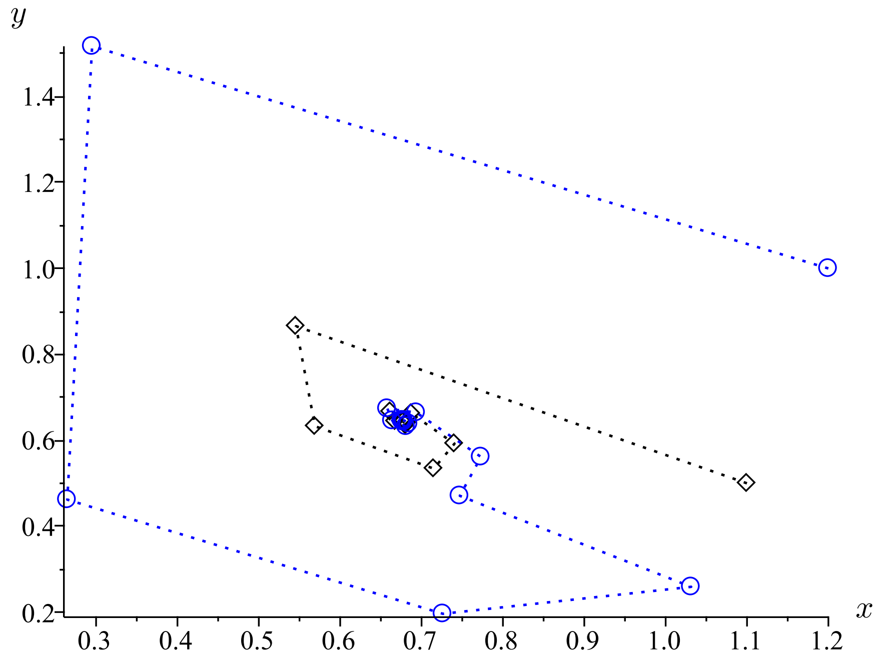
Nontrivial fixed point of (3) *vs.* r with $c = 2$. Dotted curve: x_*. Solid curve: y_*.

Note that $r \to \infty$, the number of hosts asymptotes to the carrying capacity $x_t = 1$. Similarly, the number of hosts also asymptotes to a fixed value, since it is linearly related to x_*.



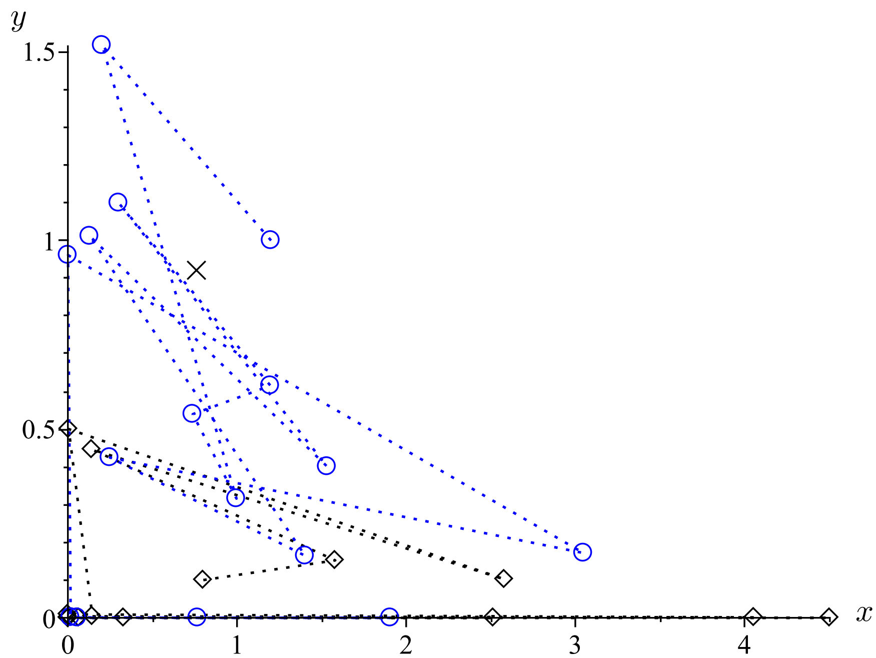
Determining stability of the nontrivial fixed point with $c = 2$.
Solid curve: $|\operatorname{tr} J|$. Dashed curve: $1 + \det J$. Here $r_* \approx 3.39$.

The stability is determined by the Jury conditions, as shown above.



Trajectories of (3) with $c = 2$, $r = 2 < r_*$. (Animation online.)

In the case where $r < r_*$, the hosts reproduce slowly enough that the fixed point is stable and the populations converge to (x_*, y_*).



Trajectories of (3) with $c = 2$, $r = 3.9 > r_*$. $\times$ marks (x_*, y_*). (Animation online.)

In the case where $r > r_*$, the fixed point is unstable, so the populations oscillate about (x_*, y_*).