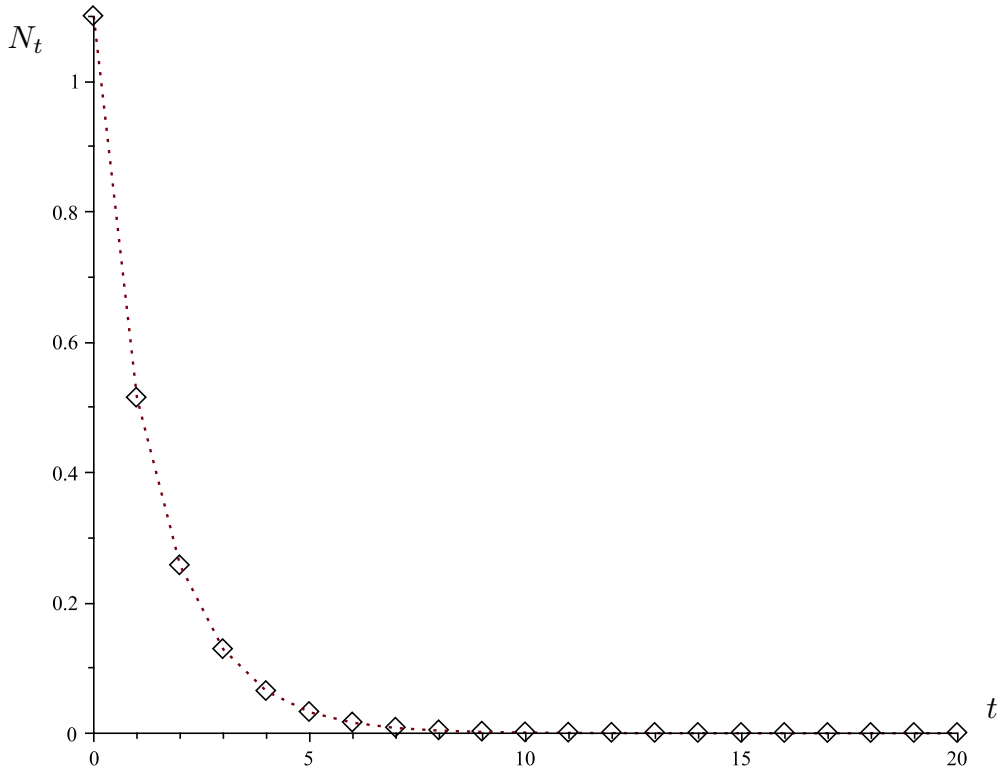
Algebraic Model (Revised)

Consider the following normalized algebraic model for insect populations:

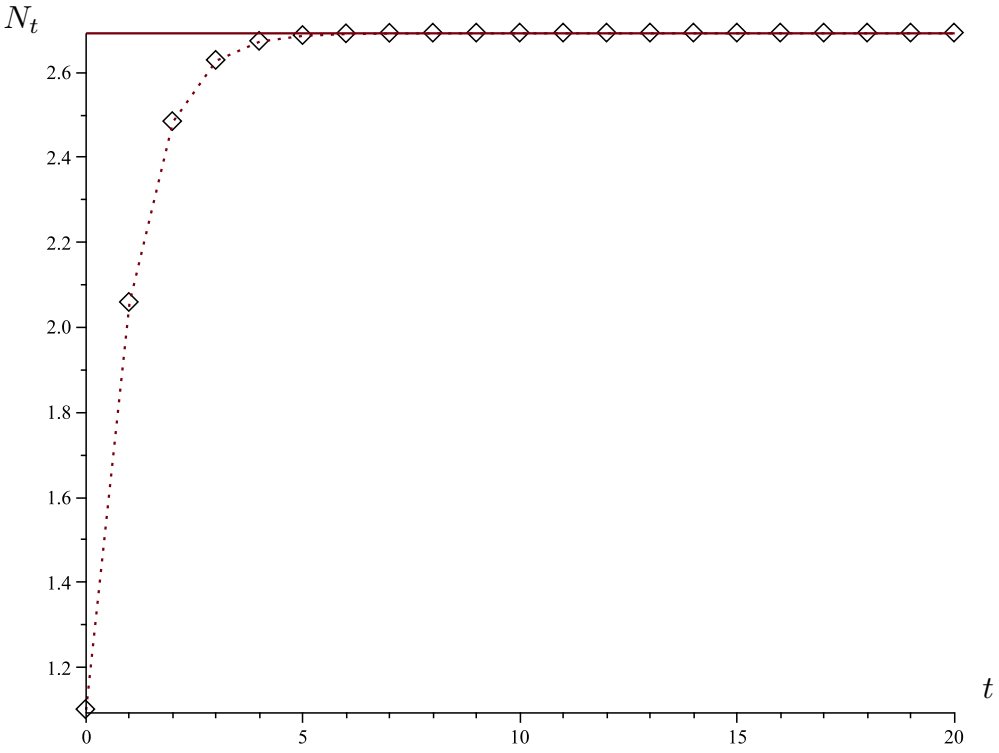
$$N_{t+1} = \lambda N_t p(N_t), \qquad p(N_t) = \begin{cases} 1, & N_t \le 1, \\ N_t^{-b}, & N_t \ge 1, \end{cases} \qquad \lambda > 0. \tag{1}$$

When $\lambda < 1$, $N_t = 0$ is the only fixed point, and it is stable:



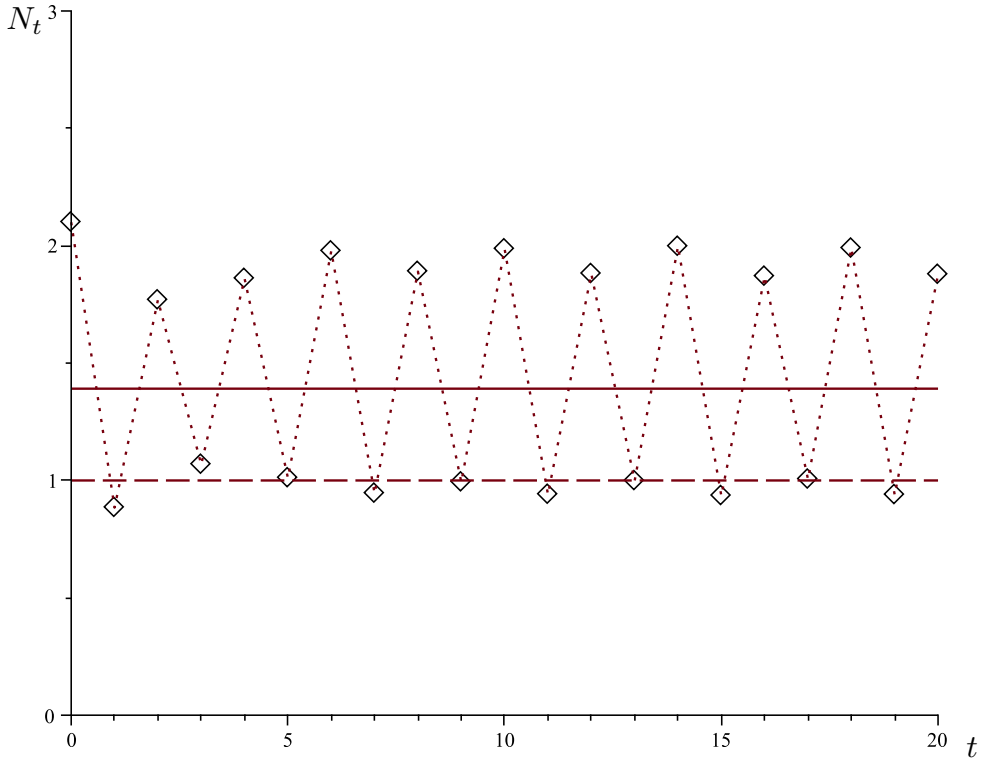
Iterations of (1) with $\lambda = 0.5$, $b = 0.7$.

When $\lambda > 1$, $N_t = 0$ is unstable and a new steady state appears at $N_* = \lambda^{1/b}$. When $0 < b < 2$, N_* is stable.



Iterations of (1) with $\lambda = 2$, $b = 0.7$. Solid line: steady state.

When $b > 2$, N_* becomes unstable and cycles appear.



Iterations of (1) with $\lambda = 2$, $b = 2.1$. Solid line: steady state.
Note the appearance of a 4-cycle.