Phase Plane Portraits

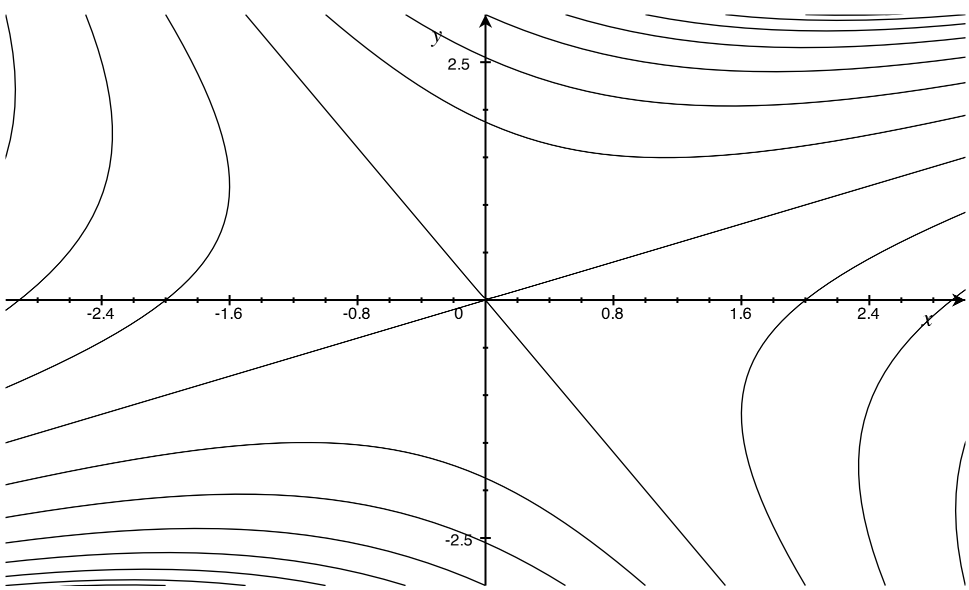
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \mathbf{x}, \tag{1}$$

the solution is

$$\mathbf{x} = c_1 e^{5t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{-5t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Since we have one positive and one negative eigenvalue, we have a saddle point, as shown below. Note the straight lines corresponding to the eigenvectors.



Phase plane of (1).

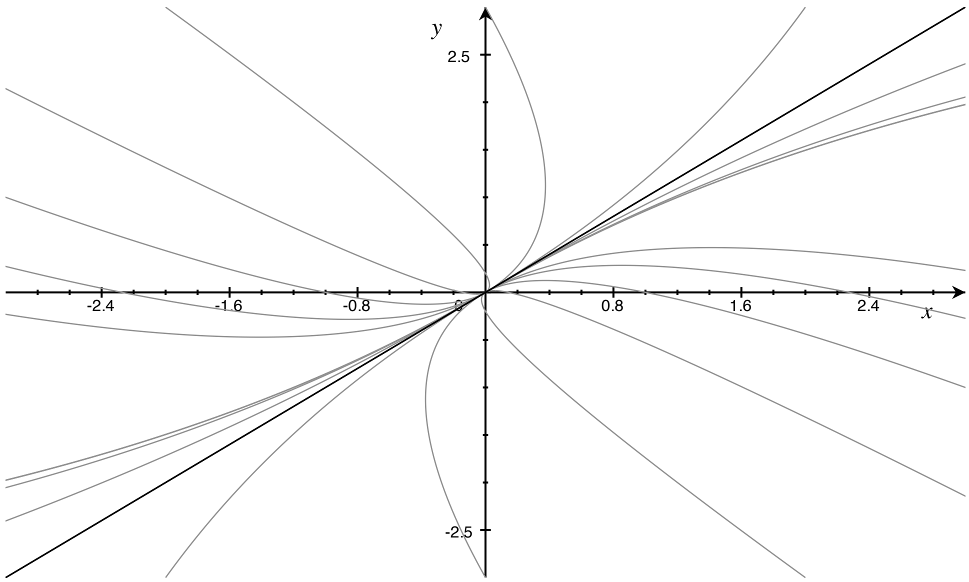
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix} \mathbf{x}, \tag{2}$$

the solution is

$$\mathbf{x} = c_1 e^{-4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Since we have two negative eigenvalues, we have a stable node, as shown below.



Phase plane of (2).

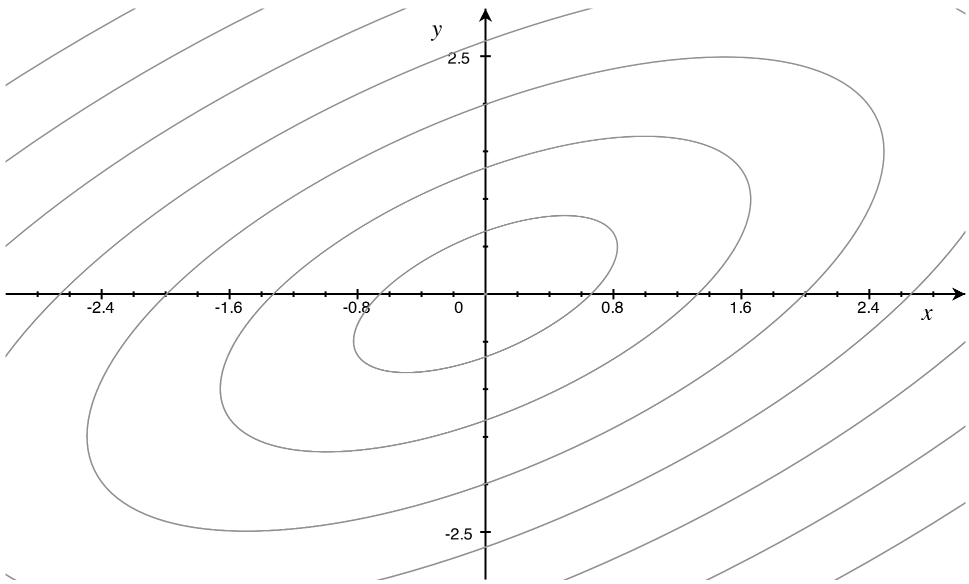
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} 3 & -5 \\ 5 & -3 \end{pmatrix} \mathbf{x}, \tag{3}$$

the solution is

$$\mathbf{x} = c_1 \begin{pmatrix} 5\cos 4t \\ 3\cos 4t + 4\sin 4t \end{pmatrix} + c_2 \begin{pmatrix} 5\sin 4t \\ 3\sin 4t - 4\cos 4t \end{pmatrix}.$$

Since there are no real eigenvectors, all trajectories spin about the origin. Since the real part of the eigenvalues is zero, the origin is a center, as shown below:



Phase plane of (3).

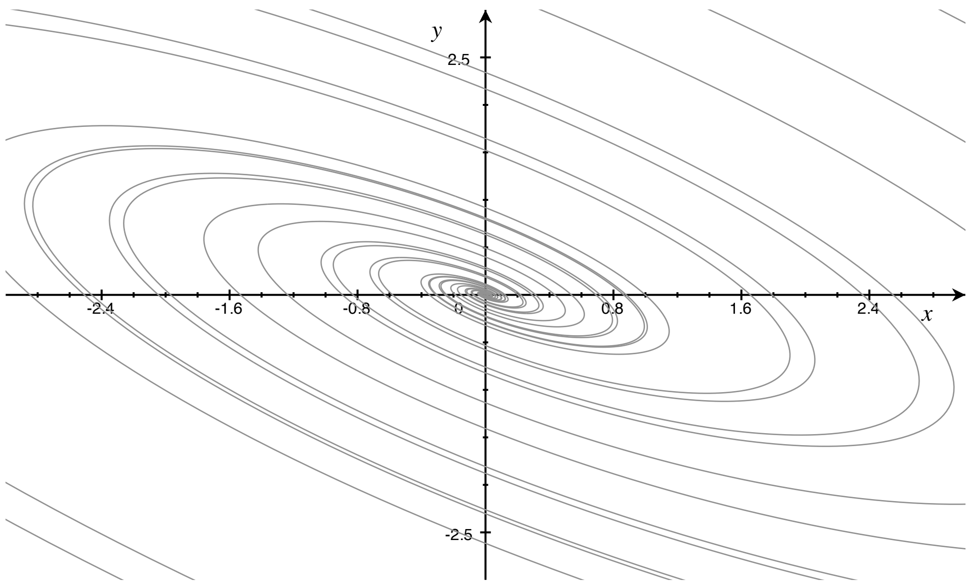
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -2 & -6 \\ 3 & 4 \end{pmatrix} \mathbf{x}, \tag{4}$$

the solution is

$$\mathbf{x} = c_1 e^t \begin{pmatrix} -\cos 3t - \sin 3t \\ \cos 3t \end{pmatrix} + c_2 e^t \begin{pmatrix} \cos 3t - \sin 3t \\ \sin 3t \end{pmatrix}.$$

Since the real part of the eigenvalues is positive, we have an unstable spiral, as shown below.



Phase plane of (4).