Updates

1. Office hours on Tuesday, 9/19 will be held from 2–3.

Homework Set 2

Read section 1.3.

Single-Species Population Dynamics

1. Suppose that in the spruce budworm model we replace the predation term by

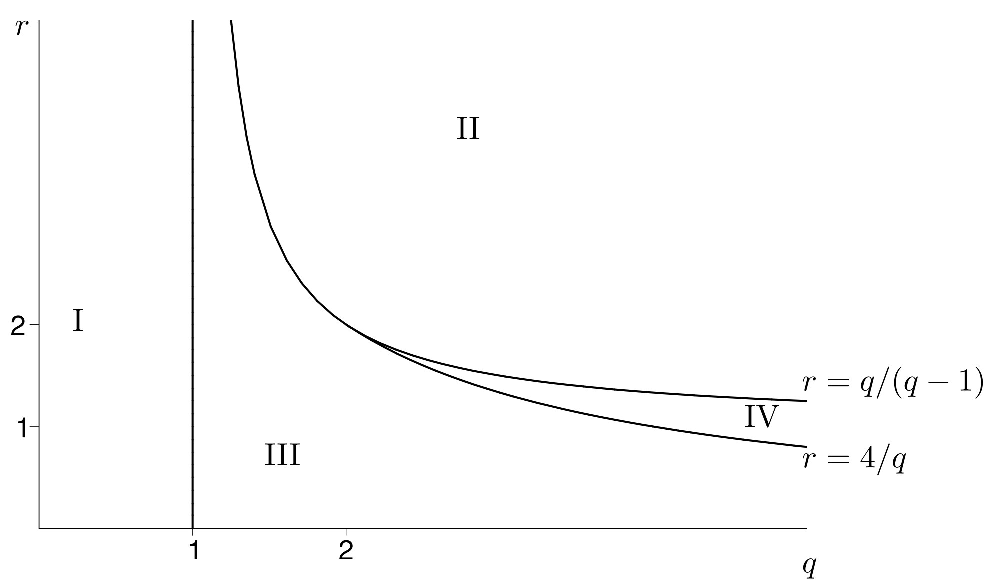
$$\tilde{P}(\tilde{N}) = \frac{B}{2}\left[1 + \tanh\left(\frac{\tilde{N} - A}{N_{\mathrm{w}}}\right)\right],$$

where $N_{\mathrm{w}} \ll A$ models the width of the transition region from no predation to full predation.

(a) (5 points) Use the same scalings as those given in class to scale the evolution equation that results.

Now let $N_{\mathrm{w}}/A = \epsilon$, where $0 < \epsilon \ll 1$.

(b) (2 points) As we take the limit $\epsilon \to 0$, what values does our dimensionless predation term take on?



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(c) (8 points) Above find a schematic of the q-r parameter plane which is divided into regions. In different regions, the number and/or (general) location of the steady states is different. Identify what is happening in each of the labeled regions, and verify that the curves listed are the appropriate boundaries of those regions.

(d) (3 points) Classify each of the steady states as stable or unstable.

2. (3 points) When a population gets small, the members may find it difficult to locate mates, causing the growth rate to be negative. This phenomenon is called the *Allee effect*. Discuss how the model

$$\dot{N} = rN\left(1 - \frac{N}{q}\right)(N - a), \qquad 0 < a < q, \tag{2.1}$$

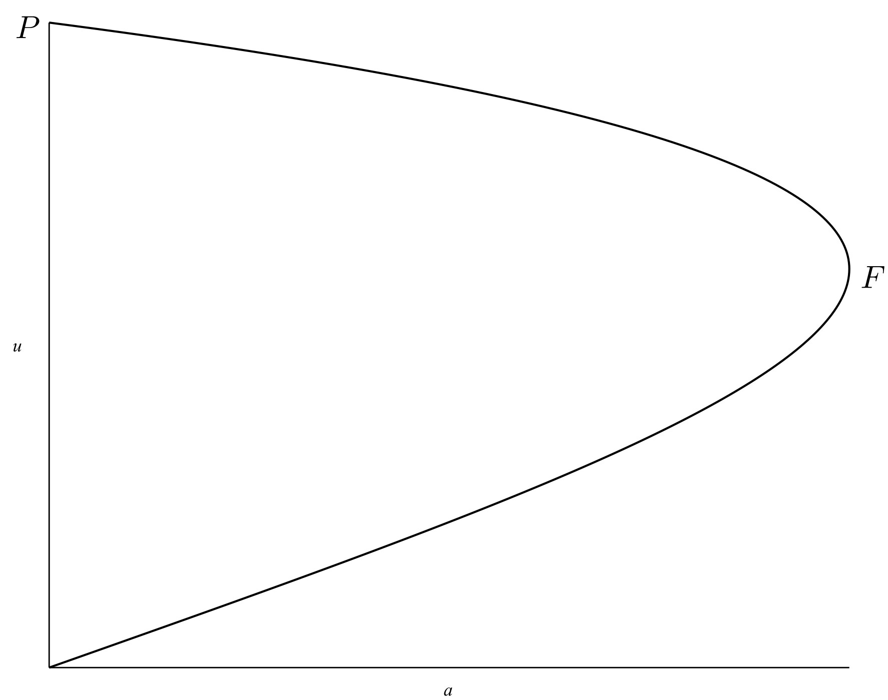
explains this effect, and what effects the additional term has on the number and stability of steady states.

Bifurcations

3. (6 points) The equation

$$\frac{du}{dt} = u(\lambda - u) - ae^{-u}. \tag{2.2}$$

has the bifurcation diagram shown below. Determine the coordinates of the intersection point P and the fold point F. (For the fold point, you should find an explicit form for u_{F}, but you may write a_{F} in terms of u_{F}.)



Bifurcation diagram of (2.2).

4. Consider the following equation:

$$\frac{du}{dt} = (u - \lambda)(u^2 - \lambda) + \epsilon. \tag{2.3}$$

(a) (5 points) Draw the bifurcation diagram for $\epsilon = 0$. Classify and identify the location of each of the bifurcation points. Label each of the solution curves, indicate the stability of each branch, and indicate the direction of the flow.

(b) (8 points) Sketch the bifurcation diagrams for $\epsilon > 0$ and $\epsilon < 0$. Indicate the stability of each branch, and indicate the direction of the flow. Do **NOT** attempt to solve for the functional forms of the curves. (*Hint: Examine what happens near the bifurcation points, and use smoothness properties.*)