Homework Set 1 (Revised)

Read sections 1.1, 1.2.

Dimensional Analysis

1. The *Navier-Stokes* equations are the governing equations for much of fluid mechanics. In dimensional form, they are given by

$$\rho \left(\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + \tilde{\mathbf{u}} \cdot \tilde{\nabla} \tilde{\mathbf{u}} \right) = -\tilde{\nabla} \tilde{p} + \mu \tilde{\nabla}^2 \tilde{\mathbf{u}},$$

where ρ is the density, $\tilde{\mathbf{u}}$ is the velocity, $\tilde{p}$ is the pressure, and μ is the bulk viscosity. The $\tilde{\nabla}$ indicates that the spatial derivatives are taken with respect to variables with dimensions.

   (a) (4 points) Write the dimensions of each of the variables listed in the Navier-Stokes equations. Check your work by verifying that each of the terms in the equation have the same dimensions.

   (b) (6 points) Given a characteristic velocity U and length scale L, scale the equations. On how many dimensionless parameters does your solution depend?

   (c) (4 points) A characteristic *inertial force* F_{i} depends on the density of the fluid ρ, the characteristic velocity U, and the characteristic length L. Calculate F_{i}. Since we are calculating a characteristic force (*i.e.*, one by which you would divide the dimensional quantities in the problem), you may set any arbitrary constants equal to 1.

   (d) (4 points) A characteristic *viscous force* F_{v} depends on the bulk viscosity of the fluid μ, the characteristic velocity U, and the characteristic length L. Calculate F_{v}.

   (e) (2 points) The *Reynolds number* Re of a system is given by the ratio of the characteristic inertial force to the characteristic viscous force. Rewrite your answer to (b) using the Re notation.

2. (4 points) Logan section 1.1, exercise 12

3. A ball of mass m is tossed upward with initial velocity V. If $\tilde{v}$ is the velocity of the ball, we assume that the force casued by air resistance is proportional to $\tilde{v}|\tilde{v}|$ and that the gravitational field is constant.

   (a) (2 points) Formulate an initial value problem for the height $\tilde{x}$ of the ball at any time $\tilde{t}$.

   (b) (4 points) Using your notes from class, choose meaningful characteristic length and time scales and recast the problem in dimensionless form. Your initial conditions should be free of all parameters.

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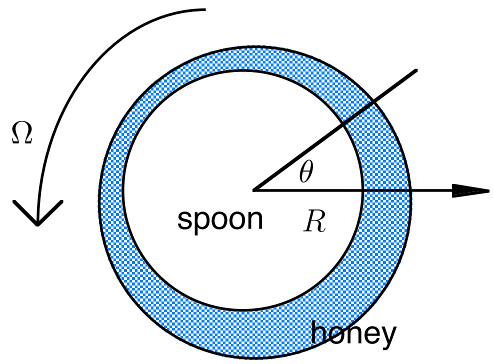

4. Suppose that a person dips the end of a wooden spoon of radius R into a viscous fluid (honey, for instance), then removes the spoon and holds it horizontal. The person then twirls the end of the spoon around with angular velocity Ω (see figure). The equation for the velocity $\tilde{u}$ of the fluid in the θ-direction is given by

$$\nu \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} = g \cos\theta, \tag{1.1}$$

where ν is kinematic viscosity and $\tilde{z}$ is distance measured in the direction normal to the surface of the spoon. Here $\tilde{z} = 0$ corresponds to the surface of the spoon.

(a) (2 points) What is the velocity of the honey at the surface of the spoon?

(b) (2 points) If H is some characteristic height of the honey, find the characteristic acceleration due to the rotation of the spoon.

(c) (2 points) Find the dimensionless parameter on which the solution must depend.

(d) (4 points) How does the solution behave as $\Omega \to \infty$? How does it behave as $\Omega \to 0$? (*Hint: How do you expect the physical system to react in these instances?*) Classify each of the steady states as stable or unstable.