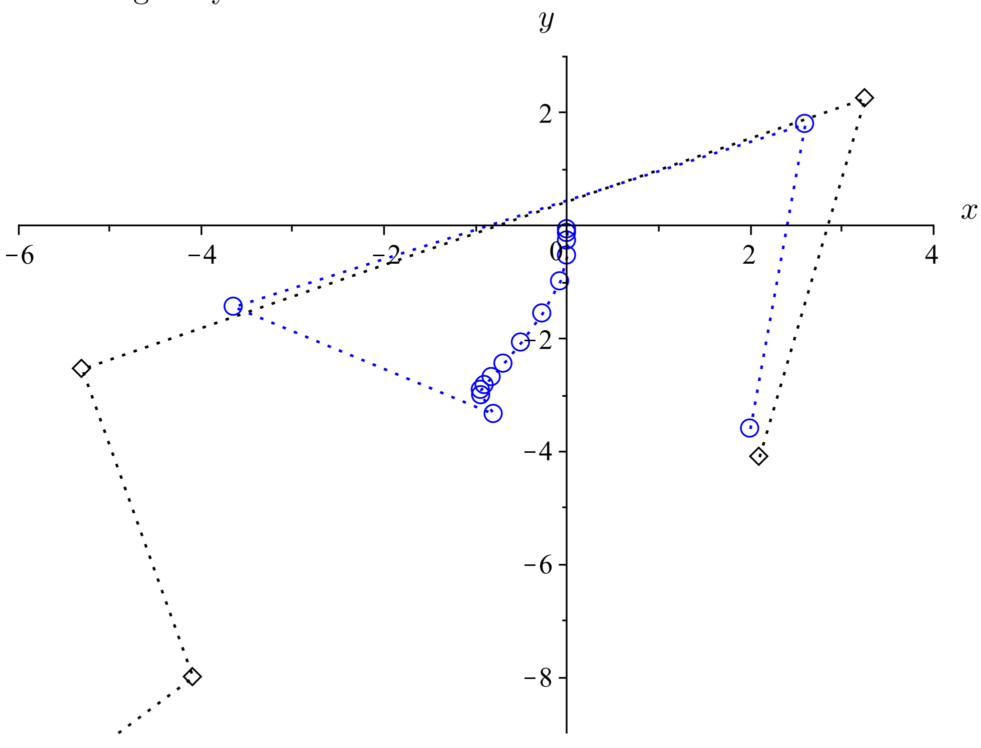
2-D Discrete Systems

Consider the two-dimensional system

$$\begin{aligned} x_{t+1} &= -\frac{x_t(y_t+1)}{2}, \\ y_{t+1} &= \frac{y_t(1-x_t)}{2}. \end{aligned} \tag{1}$$

Trajectories are shown below. Note that even though the origin is stable, the existence of the unstable fixed point at $(-1,-3)$ means that trajectories with slightly different initial conditions will have grossly different behavior.

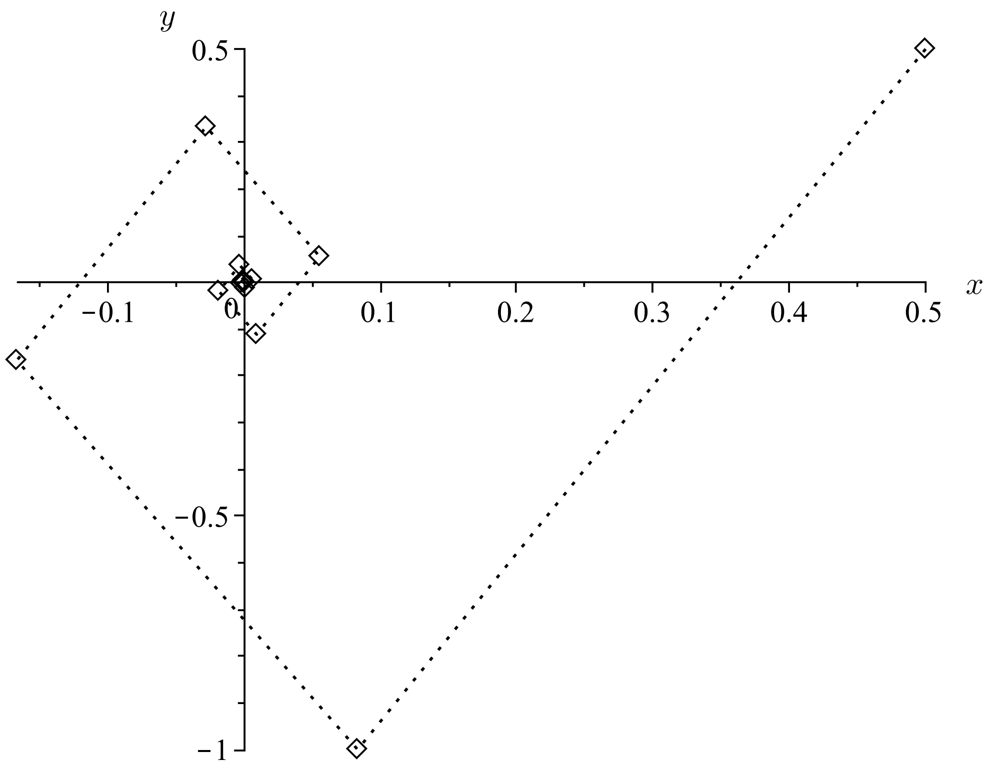


Trajectories of (1).

Consider the two-dimensional system

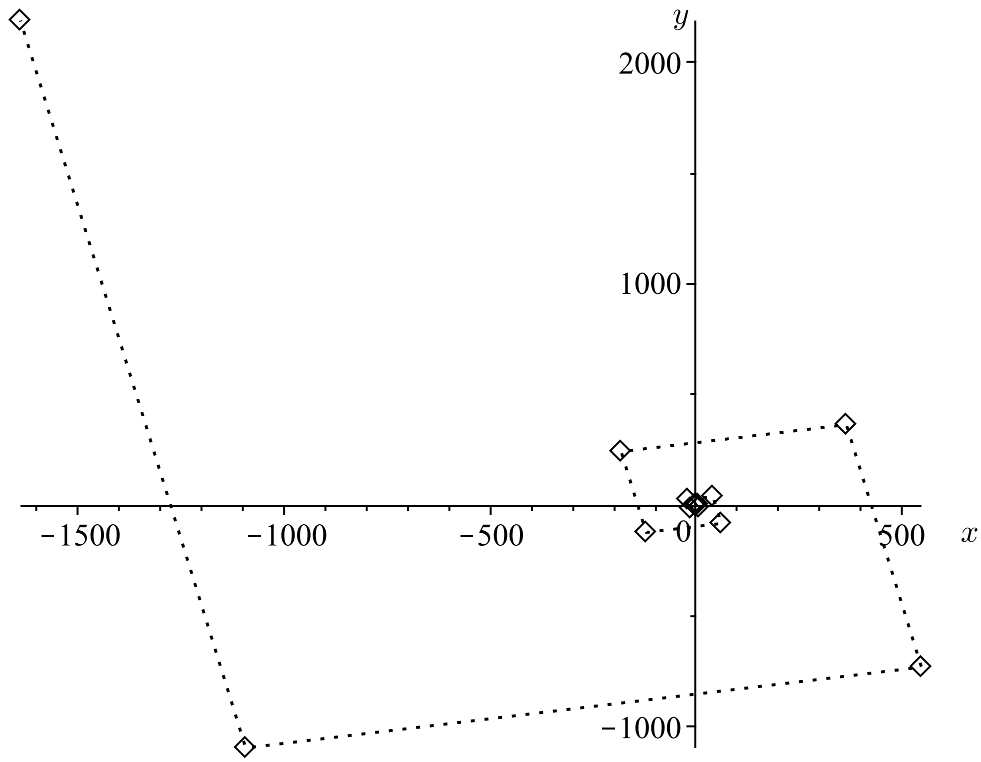
$$\begin{aligned} x_{t+1} &= \alpha y_t, \\ y_{t+1} &= -2x_t. \end{aligned} \tag{2}$$

Trajectories are shown below for various values of α.



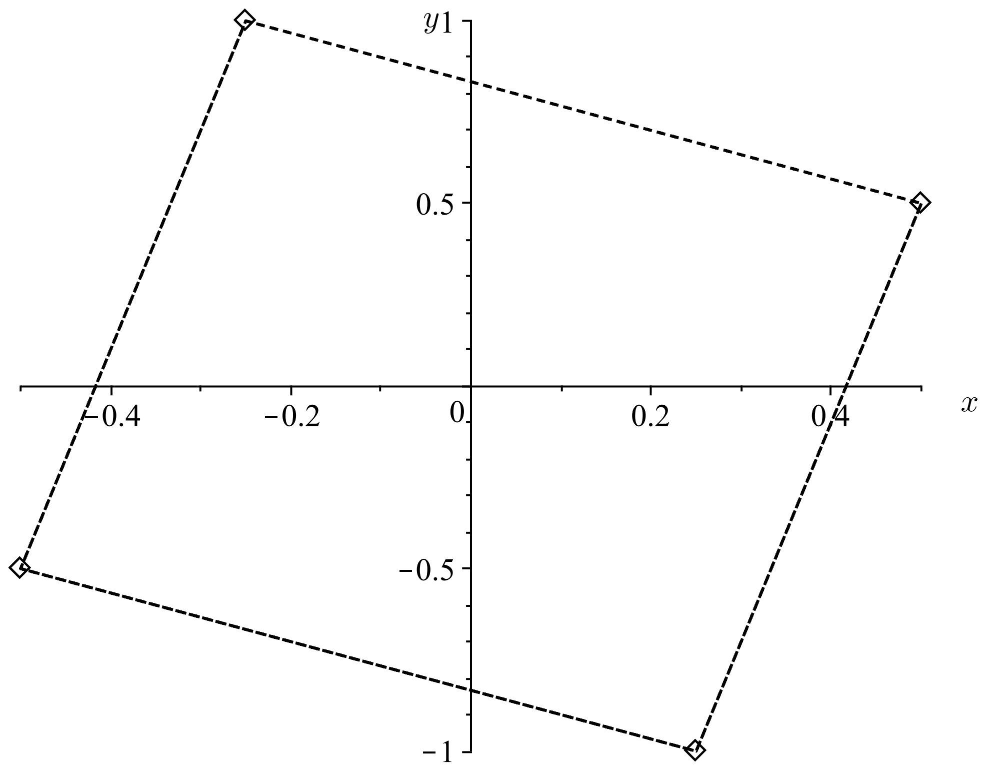
Trajectories of (2), $\alpha = 1/6$.

When $\alpha < 1/2$, the origin is stable, so this trajectory spirals in.



Trajectories of (2), $\alpha = 3/2$.

When $\alpha > 1/2$, the origin is unstable, so this trajectory spirals out.



Trajectories of (2), $\alpha = 1/2$.

When $\alpha = 1/2$, we have a 4-cycle.