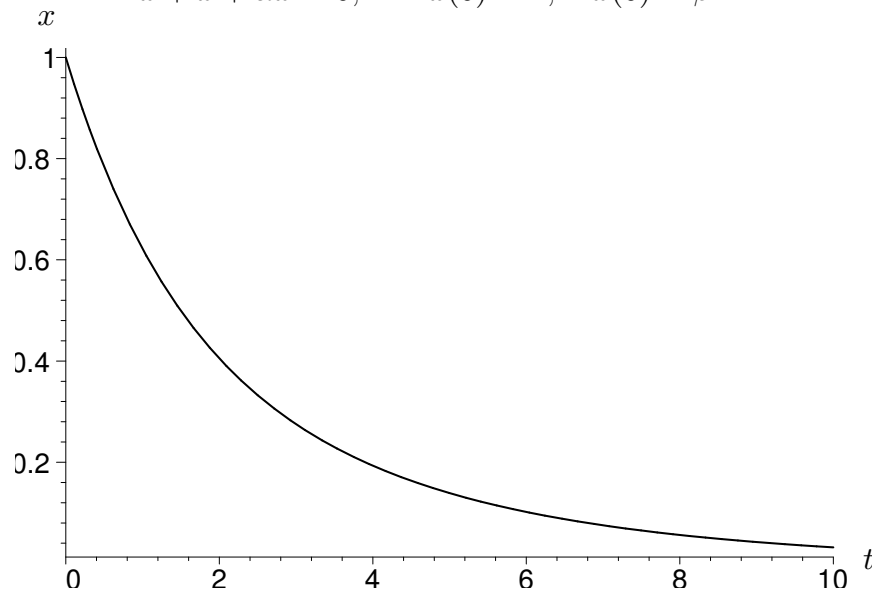


Unforced Spring System

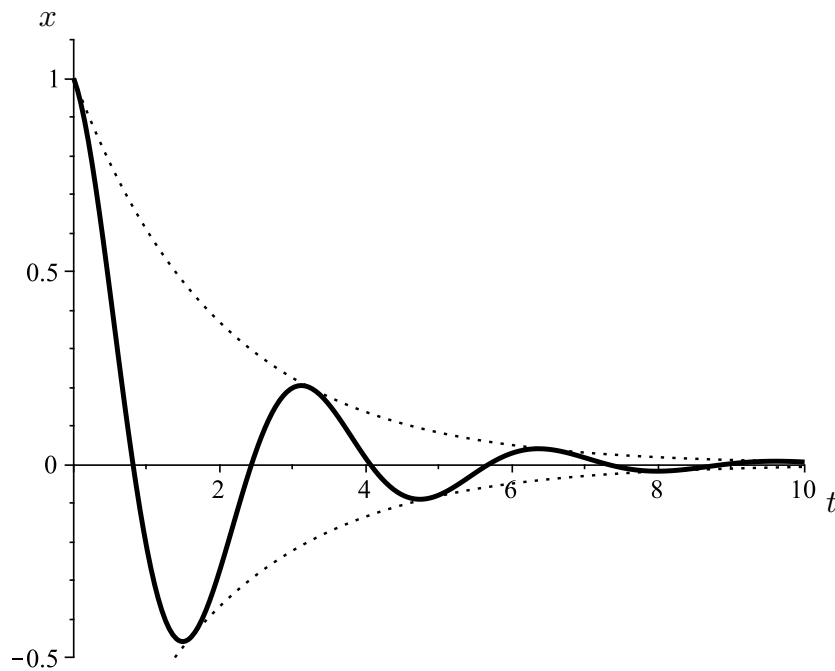
In class we derived the dimensionless spring system

$$\ddot{x} + \dot{x} + \alpha x = 0, \quad x(0) = 1, \quad \dot{x}(0) = \beta.$$



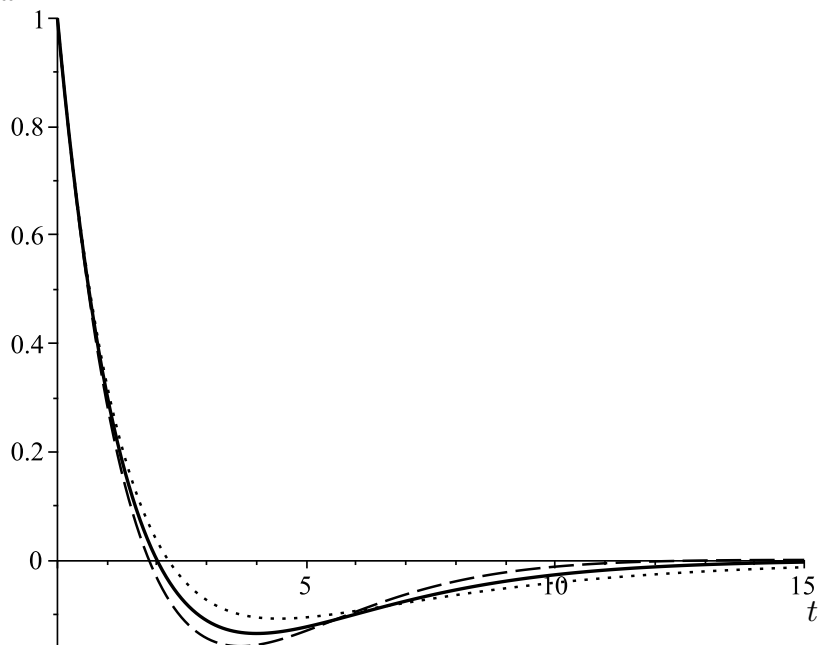
$$x(t) \text{ vs. } t \text{ for } \alpha = 0.2, \beta = -0.5.$$

In this case $\alpha < 1/4$, so the solution is overdamped.



Solid curve: $x(t)$ vs. t for $\alpha = 4$, $\beta = -0.5$. Dotted curves: envelopes $\pm e^{-t/2}$.

In this case $\alpha > 1/4$, so the solution is underdamped.



$x(t)$ vs. t for $\beta = -1$ and $\alpha = 0.18$ (dotted), 0.25 (solid), and 0.32 (dashed).

Here the solid curve is the critically damped case $\alpha = 1/4$, and the other curves show an underdamped and overdamped case with nearby values of α . Note their similarity.