

# Facts About Invertible Matrices

Let  $A \in \mathcal{R}^{n \times n}$ . Then the following statements are *equivalent*. By that I mean that if one of them is true, they are all true. If one of them is false, they are all false.

## Basic Facts

1.  $A$  is invertible.
2.  $\det A \neq 0$ .
3. 0 is not an eigenvalue for  $A$ .

## Solutions of Linear Systems

4.  $A\mathbf{x} = \mathbf{b}$  has exactly one solution for every  $\mathbf{b} \in \mathcal{R}^n$  (no free variables).
5.  $A\mathbf{x} = \mathbf{0}$  has only the trivial solution  $\mathbf{x} = \mathbf{0}$ .

## Row Reduction

6.  $A$  can be row reduced to  $I$ .
7.  $A$  is a product of elementary matrices.
8.  $A$  has  $n$  pivot columns.

## Row, Column, and Null Spaces

9. The columns of  $A$  are linearly independent.
10.  $\text{col } A = \mathcal{R}^n$ , and the columns of  $A$  form a basis for  $\mathcal{R}^n$ .
11. The rows of  $A$  are linearly independent.
12.  $\text{row } A = \mathcal{R}^n$ , and the rows of  $A$  form a basis for  $\mathcal{R}^n$ .
13.  $\text{rank } A = n$ .
14.  $\mathcal{N}(A) = \{\mathbf{0}\}$ .
15.  $\dim \mathcal{N}(A) = 0$ .

