Updates

Exam II will be administered on Monday, Oct. 31. It will cover up through section 4.2. You will need a small blue book.

Homework Set 8

Read sections P3.5, 6.1–6.3.

Section P3.5/6.1

1. Show that the set P of all polynomials, with the normal definitions of addition and scalar multiplication, forms a vector space.
2. Let A be a particular vector (matrix) in $\mathcal{R}^{2\times 2}$. Determine whether or not the following are subspaces of $\mathcal{R}^{2\times 2}$:
   (a) $S_1 = \{B \in \mathcal{R}^{2\times 2} | AB = BA\}$.
   (b) $S_2 = \{B \in \mathcal{R}^{2\times 2} | AB \neq BA\}$.
   (c) $S_3 = \{B \in \mathcal{R}^{2\times 2} | BA = O\}$.
3. Let
$$A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & -2 & 4 \\ 3 & -7 & 10 \end{pmatrix}.$$
   Calculate $\mathcal{N}(A)$ and $\mathcal{N}(A^T)$.

Section P3.5/6.2

4. Let $C[0, 1]$ be the space of continuous functions defined for $0 \leq x \leq 1$. For each of the following, consider the subspace V of $C[0, 1]$ spanned by the set of functions. Find a basis for V and the dimension of V.
   (a) x, $x^2 + 1$, $x^2 - 1$
   (b) $\sin x \cos x$, $\sin 2x$
   (c) x, $\sin x$, $\cos x$
   (d) e^x, e^{-x}, $\sinh x$

5. Consider the vectors

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{x}_3 = \begin{pmatrix} 9 \\ -1 \\ -2 \end{pmatrix}, \quad \mathbf{x}_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

(a) What is the dimension of $\text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$?
(b) Is the set $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$ linearly independent? Justify your answer.
(c) Identify which subsets of $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$ form bases of $\mathcal{R}^3$.

6. Find the dimension of and a basis for each of the vector spaces listed. (*Hint: Factor out the variables.*)

$$\text{(a)}: \quad \begin{pmatrix} 2a - b \\ a + b \\ b \\ -a + 3b \end{pmatrix}, \qquad a, b \in \mathcal{R}$$

$$\text{(b)}: \quad \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}, \qquad a, b \in \mathcal{R}$$

Section P3.5

7. For each of the following matrices, find a basis for row A, col A, and $\mathcal{N}(A)$.

(a)

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix}$$

8. Consider the following matrix and vector:

$$A = \begin{pmatrix} 3 & -1 \\ -6 & 2 \\ -3 & 1 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} -4 \\ 8 \\ 4 \end{pmatrix}.$$

(a) Is $\mathbf{b} \in \text{col}\, A$?
(b) Is the system $A\mathbf{x} = \mathbf{b}$ consistent? If so, how many solutions are there?

Section P6.3

9. Let $B = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n]$ and $F = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n]$ be two ordered bases for $\mathcal{R}^n$ and let $U = (\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n)$ and $V = (\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n)$ be two $n \times n$ matrices whose columns are the basis vectors in B and F, respectively. Show that the transition matrix ${}_{B\leftarrow F}P$ from F to B can be determined by calculating the reduced row echelon form of $(U|V)$.

10. Consider the two ordered bases $B = [1, 1+x, 1+x+x^2]$ and $C = [1, 1-x, 1-x^2]$ for $\mathcal{P}_2$.

    (a) Find the transition matrix ${}_{B\leftarrow C}P$.
    (b) Calculate $\mathbf{b} = [2x^2 + x]_B$ and $\mathbf{c} = [2x^2 + x]_C$.
    (c) Verify that $\mathbf{c} = {}_{C\leftarrow B}P\,\mathbf{b}$.