MATH 351-010 Engineering Mathematics I
Prof. D. A. Edwards Due: Oct. 28, 2022

# Updates

1. The final exam for this course has been scheduled for Tuesday, Dec. 13 from 10:30–12:30 in **GOR 102**.
2. A review session for the final exam will be held on Monday, Dec. 12 from 11:30–1:30 in **PRN 227**.

# Homework Set 7

Read sections P2.3 and P4.2.

## Section P4.2

1. page 282, exercise 54
2. Calculate $\det B$, where
$$B = \begin{pmatrix} 0 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 \\ 1 & 2 & 2 & 1 \\ -1 & -1 & 3 & 7 \end{pmatrix}.$$
(*Hint: At each stage, pick the row or column that produces the easiest expression.*)
3. Consider the following matrix in $\mathcal{R}^{5\times 5}$:
$$A = \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{pmatrix},$$
where the $*$s are arbitrary entries. Show that $\det A = 0$.
4.
   (a) Calculate the determinant of
$$\begin{pmatrix} 1-\lambda & 3 \\ 5 & 3-\lambda \end{pmatrix}.$$
   (b) Find the value(s) of $\lambda$ such that the determinant is 0.
5. Let $A \in \mathcal{R}^{n\times n}$, and let the first and second rows of $A$ be equal.
   (a) Calculate the determinant by expanding along each of the first and second rows.
   (b) Show that $\det A = 0$.

6. Suppose that $A$, $B$, and $C$ are $n \times n$ matrices such that $\det A = 2$, $\det B = -5/3$, and $\det C = 0$. Calculate the following determinants:
   (a) $\det A^{-1}$
   (b) $\det(3B)$
   (c) $\det(ABC)$
   (d) $\det(A^T B^{-1})$

7. Let
$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}.$$
Show by direct calculation that $\det AB = (\det A)(\det B)$.

## Section P2.3

8. Determine whether each of the following sets is linearly independent in the appropriate vector space:

   (a) $$\left\{ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 11 \end{pmatrix} \right\}$$
   (b) $$\left\{ \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 3 & 4 \end{pmatrix} \right\}$$
   (c) $$\left\{ \sin^3 x, \sin 3x, \sin x \right\}$$

9. (BH) Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in a vector space $V$. Prove that $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is also linearly independent, where $\mathbf{w}_1 = \mathbf{v}_1 - \mathbf{v}_2 + 2\mathbf{v}_3$, $\mathbf{w}_2 = \mathbf{v}_2 - \mathbf{v}_3$, and $\mathbf{w}_3 = \mathbf{v}_3$.

10. (BH) Consider the following vectors:
$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{x}_3 = \begin{pmatrix} -3 \\ z \\ 9 \end{pmatrix}.$$
   (a) For which value(s) of $z$ does the equation $\mathbf{x}_3 = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$ have a solution?
   (b) For which value(s) of $z$ are $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ linearly dependent?