MATH 351-010 | Engineering Mathematics I
Prof. D. A. Edwards | Due: Oct. 21, 2022

Homework Set 6

Read sections P3.1–3.3.

Sections P3.1/3.2

1. Consider the following matrices:

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 3 \\ 2 & -3 \\ -4 & -1 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & -1 \\ 6 & -3 \end{pmatrix}.$$

   If possible, compute

   (a) $A(BC)$
   (b) $(AB)C$

2. Consider the following matrices:

$$D = \begin{pmatrix} 1 & -2 & 1 \\ 2 & -2 & 3 \end{pmatrix}, \qquad E = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 1 \\ -3 & 1 & -2 \end{pmatrix}, \qquad F = \begin{pmatrix} 1 & 0 & 2 \\ -3 & 3 & -3 \\ 2 & 1 & 4 \end{pmatrix}.$$

   If possible, compute

   (a) $D(E+F)$
   (b) $DE + DF$

3. Let $A \in \mathcal{R}^{n\times n}$ and let $B = A + A^T$, $C = A - A^T$.

   (a) Show that B is symmetric and C is antisymmetric. (A matrix C is *antisymmetric* if $C = -C^T$.)
   (b) Show that every $n \times n$ matrix can be represented as a sum of a symmetric matrix and an antisymmetric matrix.

4. Consider the linear system

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &= 3, \\ 4x_1 - x_2 - x_3 &= 3, \\ -x_1 + 7x_2 + x_3 &= 0. \end{aligned} \tag{6.1}$$

   (a) Write the system (6.1) as a matrix-vector equation.
   (b) Solve the system (6.1).

5. Consider the following matrices:

$$A = \begin{pmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{pmatrix}.$$

   (a) Verify that $A+B = B+A$, $3(A+B) = 3A+3B$, and $(A+B)^T = A^T + B^T$.
   (b) Calculate the following (or indicate the product doesn't exist):

$$AB, \qquad A^T B, \qquad A^T B^T.$$

6. Let

$$A = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}.$$

   (a) Compute $(AB^T)C$.
   (b) Compute $B^T C$ and multiply the result by A on the right.
   (c) Explain why $(AB^T)C = (B^T C)A$.

Section P3.3

7. Show that if A is a symmetric nonsingular matrix, then A^{-1} is also symmetric.

8. Let

$$C = \begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

   (a) Calculate C^{-1}.
   (b) Use your answer to (a) to show that the solution of $C\mathbf{x} = \mathbf{b}$ for the following cases is the vector listed:

$$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \implies \mathbf{x} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}; \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \implies \mathbf{x} = \begin{pmatrix} 2 \\ -8 \\ 2 \end{pmatrix}.$$

9. For each pair of matrices $\{A, B\}$, find an elementary matrix E such that $EA = B$. Also, explain in words what row operations each elementary matrix performs.

$$A_1 = \begin{pmatrix} 1 & 4 \\ 0 & 3 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 0 & 3 \\ 1 & 4 \end{pmatrix}, \tag{a}$$

$$A_2 = \begin{pmatrix} 2 & -3 \\ 3 & 1 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 2 & -3 \\ -9 & -3 \end{pmatrix}, \tag{b}$$

$$A_3 = \begin{pmatrix} 2 & 4 & 7 \\ -1 & 4 & 2 \\ 2 & 1 & 1 \end{pmatrix}, \qquad B_3 = \begin{pmatrix} 2 & 4 & 7 \\ 0 & 9 & 5 \\ 2 & 1 & 1 \end{pmatrix}. \tag{c}$$

10. Consider the following matrix:

$$C = \begin{pmatrix} 1 & \alpha & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ 1 & \alpha & 1 & \alpha \\ 1 & -1 & \alpha & 1 \end{pmatrix}.$$

Show that C is not invertible *only if* $\alpha = \pm 1$.