MATH 351-010 Engineering Mathematics I
Prof. D. A. Edwards Due: Oct. 14, 2022

Updates

Exam I will be administered on Monday, Oct. 3. It will cover up through Homework Set 4. You will need a small blue book.

Homework Set 5 (Revised)

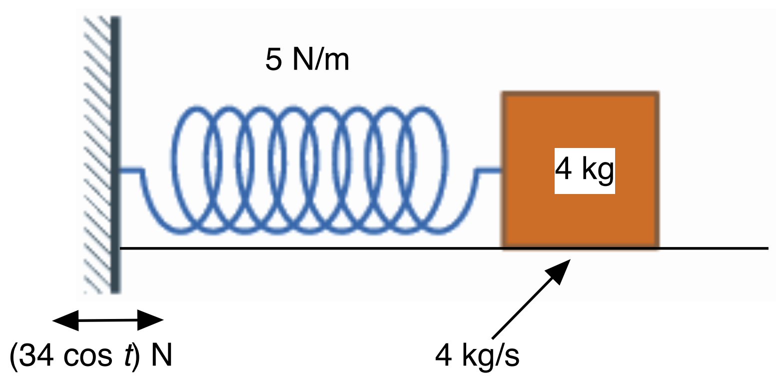
Read sections Z3.5, Z3.8, P2.1, P2.2, P3.1, P3.2.

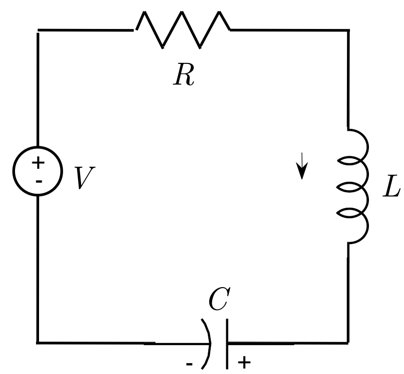
Section Z3.5

(For the problems in this section, use the method of variation of parameters.)

1. Consider the equation
$$y'' + y = \csc^2 x.$$
   (a) Solve for a particular solution using the method of variation of parameters.
   (b) Where is this equation guaranteed to have a unique solution?

2. Find the general solution of
$$\ddot{y} - 4\dot{y} + 4y = \frac{e^{2t}}{t}.$$





Section Z3.8(b)

3. We reconsider the spring system of Homework Set 3, #5. As before, the spring has stiffness $k = 5$ N/m, is damped with damping constant $b = 4$ kg/s, and is attached to a weight with mass $M = 4$ kg. However, now we impose the following discontinuous oscillation of the support (in Newtons; see figure above left):

$$F(t) = \begin{cases} 34\cos t, & 0 \le t \le 2\pi, \\ 0, & t > 2\pi. \end{cases}$$

Initially the spring is extended 2 m and given a velocity of -2 m/s.

(a) Solve the resulting system for the displacement $x(t)$ in the region $t \in [0, 2\pi]$.

(b) Show that

$$x(2\pi) = 2, \qquad \dot{x}(2\pi) = 8 - 10e^{-\pi}. \tag{5.1}$$

(c) Using the fact that x and $\dot{x}$ should be continuous at $t = 2\pi$, calculate the displacement x in the region $t > 2\pi$.

4. We reconsider the *series RLC* circuit of Homework Set 4, #4, but now we apply an AC voltage of $B\cos\omega t$ (see figure above right). There is an initial voltage on the capacitor of V_0 and initially there is a current I_0 in the inductor.

(a) Explain in words why the governing equations for this system are

$$L\ddot{I} + R\dot{I} + \frac{I}{C} = -B\omega\sin\omega t, \qquad I(0) = I_0, \qquad \dot{I}(0) = -\frac{V_0 + RI_0}{L}. \tag{5.2}$$

(b) Show that in the case where $L = 1$, $C = 1/5$, and $R = 4$, the steady state for the system is

$$I_s(t) = \frac{B\omega[4\omega\cos\omega t + (\omega^2 - 5)\sin\omega t]}{\omega^4 + 6\omega^2 + 25}. \tag{5.3}$$

Section P2.1

5. An oil refinery produces low-sulfur and high-sulfur fuel.
   - Producing one ton of low-sulfur fuel l takes 10 minutes in the blending plant and 8 minutes in the refining plant.
   - Producing one ton of high-sulfur fuel h takes 8 minutes in the blending plant and 4 minutes in the refining plant.
   - The blending plant is available for 8 hours a day, and the refining plant is available for 6 hours a day.

   (a) Write the system of equations needed to solve for the tons of l and h necessary to utilize the plant fully.

   (b) Show (by deriving the solution, **NOT** direct substitution) that we can make 40 tons of l and 10 tons of h.

6. You are given three alloys of the following composition:

   Alloy A: 5 parts (by weight) gold, 2 silver, 1 lead
   Alloy B: 2 parts gold, 5 silver, 1 lead
   Alloy C: 3 parts gold, 1 silver, 4 lead

   (a) How much of each metal is in one ounce of Alloy A?

   Suppose we want to make 27 ounces of a new alloy containing equal quantities (by weight) of gold, silver, and lead.

   (b) Write the system of equations needed to solve for the amounts of A, B, and C necessary to make this new alloy.

   (c) Show (by deriving the solution, **NOT** direct substitution) that we need 15 ounces of C, 11 ounces of B, and 1 ounce of A.

7. Suppose that the following facts are true:

   (1) Of the number of people x_{s} who start a year living on the East Coast, 80% stay in and 20% move out during the course of that year.

   (2) Of the number of people y_{s} who start a year living off the East Coast, 90% stay out and 10% move in during the course of that year.

   (3) The number of births and deaths cancel one another out (so you don't have to worry about them).

   Let x_{e} and y_{e} be the number of people living on and off the East Coast, respectively, at the *end* of the year.

   (a) Write the system of equations needed to solve for x_{e} and y_{e} as functions of x_{s} and y_{s}.

   The system in (a) is an example of a *Markov* process, and we will return to study such systems later.

   (b) If $y_{\mathrm{e}} = 155$ million and $x_{\mathrm{e}} = 95$ million, find x_{s} and y_{s}.

   (c) If $x_{\mathrm{s}} = x_{\mathrm{e}}$ and $y_{\mathrm{s}} = y_{\mathrm{e}}$, show that the ratio $y_{\mathrm{e}}/x_{\mathrm{e}} = 2$.

   In the case given by (c), $(x_{\mathrm{s}}, y_{\mathrm{s}})$ is called an *eigenvector* of the linear system. We will study this concept in great detail later.

Section P2.2

8. Consider the system

$$\begin{aligned} x - \ \ 3y + \ \ \ \ z &= 1 \\ 2x + \ \ 2y - \ \ 3z &= 2 \\ -5x - 17y + 15z &= b, \end{aligned} \tag{5.4}$$

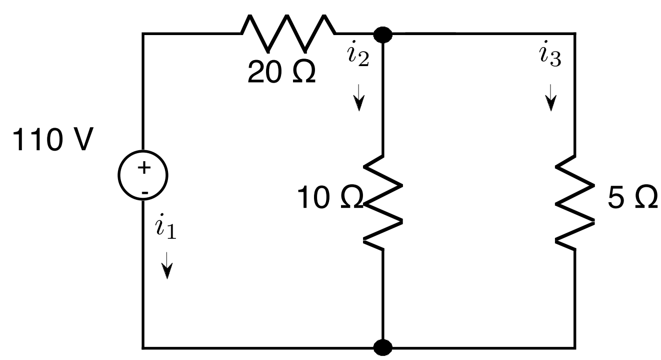
where b is a constant.

(a) Write the system in augmented matrix form, and row reduce it to the following form:

$$\begin{pmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & * & * \end{pmatrix},$$

where the * are unknown entries that you must find.

(b) Find all values of b for which (5.4) has a solution, and write the solution in that case.



9. Solve for the currents in the network above.

Section P3.1/3.2(a)

10. Let $A \in \mathcal{R}^{m\times n}$, $\mathbf{b} \in \mathcal{R}^m$, $\mathbf{x} \in \mathcal{R}^n$. Moreover, let $A\mathbf{x} = \mathbf{b}$, and $\mathbf{z} = 3\mathbf{b}$. Does the equation $A\mathbf{y} = \mathbf{z}$ have a solution? If so, compute it. If not, explain why not.