MATH 351-010 Engineering Mathematics I
Prof. D. A. Edwards Due: Sept. 16, 2022

Homework Set 2

Read sections Z1.2, Z2.2, Z2.7, Z2.8, Z3.1.

Sections Z1.2, Z2.2

1. Write the general solution of the equation

$$\dot{y} = y^3 \sin t.$$

2. Write the solution of the equation

$$(t^2 + 1)\dot{y} = -(1 + y^2), \qquad y(0) = 1.$$

3. **WITHOUT** solving the problem, determine the interval in t in which the solution of

$$(t - 3)\dot{y} + (\log 4t)y = -t, \qquad y(1) = 3$$

is guaranteed to exist. Is the interval the same if the boundary condition is changed to

$$y(5) = 3?$$

4. Consider the equation

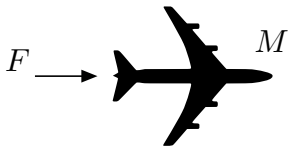
$$\dot{y} - y^5 = 0, \qquad y(0) = y_0 \neq 0.$$

(a) Write down the solution to the equation.
(b) How does the interval of existence for the solution depend on y_0?

Sections Z2.7, Z2.8

5. Beginning at time $t = 0$, cigarette smoke containing 4% carbon monoxide (CO) is introduced into a room containing 50 m^3 of air at the rate of 0.01 m^3/min, and the well-circulated mixture leaves the room at the same rate.
   (a) If $C(t)$ is the concentration of CO in the air at time t, show that
   $$\frac{dC}{dt} + (2 \times 10^{-4})C = 8 \times 10^{-6}. \tag{2.1}$$
   What is the initial condition?
   (b) Calculate $C(t)$.
   (c) Extended exposure to CO is harmful to the human body at levels of 0.012% or above. At what time does the concentration in the room reach that level?
6. According to Newton's Law of Cooling, the rate of change of temperature θ of an object is proportional to the temperature differential between it and the outside environment (θ_∞).
   (a) Explain in words why Newton's Law may be written as
   $$\dot{\theta} = -k(\theta - \theta_\infty),$$
   where k is a constant. Physically, what must the sign of k be?

   A pork loin initially at a temperature of 45° F is placed in a 425° F oven to cook. After 1 hr, the roast is 125° F.
   (b) Calculate k for this example.
   (c) How much longer should the roast cook to be cooked medium (160° F)?
   (d) Given your physical intuition, why is Newton's Law not a good model for this problem? (Usually Newton's Law is applied only to *surface* temperatures.)



7. Consider an airplane of mass M moving horizontally through the air under a constant force F (see diagram). If the horizontal velocity is given by V, the frictional force is proportional to V^2 with constant k.
   (a) Write a first-order ODE for V that describes this system. Be sure to explain the sign of each term.
   (b) Suppose that $F = k$, and the airplane starts from rest. Show that the solution is given by
   $$V(t) = \frac{1 - e^{-2kt/M}}{1 + e^{-2kt/M}}. \tag{2.2}$$
   (c) What happens to the solution (2.2) as $t \to \infty$? Explain your answer physically.

Section Z3.1(a)

8. Consider the differential equation

$$2\ddot{y} + 7\dot{y} + 3y = 0.$$

   (a) Find the general solution. Describe the long-time behavior.
   (b) Calculate the specific solution for $y(0) = 3$, $\dot{y}(0) = -4$.

9. Write down *all* equations of the form $a\ddot{y} + b\dot{y} + cy = 0$ such that the solution y approaches a multiple of e^{-3t} as $t \to \infty$.

10. Consider the following system of coupled first-order ODEs:

$$3\dot{x} + x - 4\dot{y} - 3y = 0, \tag{2.3a}$$
$$-2x + \dot{y} + 4y = 0. \tag{2.3b}$$

    (a) Eliminate x from the system to obtain a second-order ODE for y.
    (b) Show that the general solution for y is

$$y(t) = c_1 e^{t/3} + c_2 e^{-2t},$$

    and find the corresponding general solution for x.