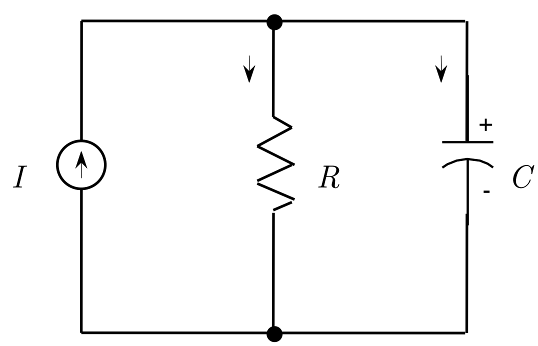
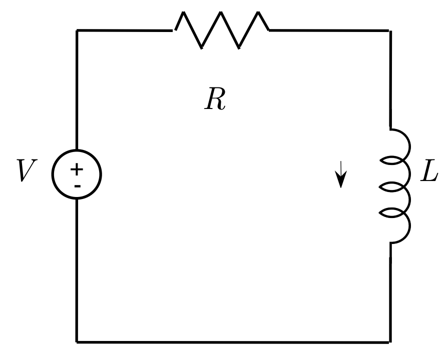
MATH 351-010 | Engineering Mathematics I
Prof. D. A. Edwards | Due: Sept. 9, 2022

Homework Set 1 (9/6 Revision)

Read sections Z1.1, Z2.3.





Section Z1.1

1. Determine by inspection two solutions of

$$(y')^2 + y^2 = 1. \tag{1.1}$$

(*Hint: Does (1.1) look like an identity you may have seen before?*)

2. Consider the driven RC circuit shown above left. The current through the resistor is given by $I_R = V/R$, where V is the voltage, and the current through the capacitor is given by $I_C = C\dot{V}$. Let the driving current $I = A\cos\omega t$, where A is the constant amplitude and ω the constant frequency. (The directions of the currents are indicated by the arrows.) The sum of the currents into the node above the resistor must be zero; use this fact to obtain the differential equation

$$\dot{V} + \frac{1}{RC}V = \frac{A\cos\omega t}{C}.$$

(*Hint: Note one of the arrows points opposite to the others.*)

3. Consider the driven RL circuit shown above right. The voltage through the inductor is given by $V_L = L\dot{I}$, where I is the current. Let the driving voltage $V = A\cos\omega t$, where A is the constant amplitude and ω the constant frequency. The sum of the voltage around this loop must be zero; use this fact to obtain the differential equation

$$\dot{I} + \frac{R}{L}I = \frac{A\cos\omega t}{L}.$$

(*Hint: What is the true sign of the imposed voltage?*)

4. A ball of mass M is driven horizontally through a fluid by a constant force F. The frictional force is proportional to the horizontal velocity V with proportionality constant k. Show that the governing equation for V is given by

$$M\dot{V} + kV = F.$$

Be sure to explain the sign of each term.

5. Consider the equation

$$4\dot{y} = -2y + 3. \tag{1.2}$$

(a) Show by direct substitution that a solution of the form

$$y(t) = A + Be^{\lambda t}. \tag{1.3}$$

satisfies (1.2). Calculate exact values for as many of the constants $\{A, B, \lambda\}$ as possible. Which terms in equation (1.2) help you determine which constants?

Now suppose that in addition to (1.2),

$$y(0) = 1. \tag{1.4}$$

(b) Use (1.4) to calculate exact values for all the constants in the proposed solution (1.3).

Section Z2.3

6. page 60, exercise 30

7. Consider the differential equation

$$\dot{y} - 3y = e^{-t}, \qquad y(0) = y_0.$$

(a) Find the solution for any y_0.
(b) Describe how the long-time behavior of y varies with y_0.
(c) Find the critical value of y_0 which separates the two types of behaviors.
(d) Describe the long-time behavior of y for that specific value of y_0.

8. Solve the differential equation

$$t\dot{y} - 4(t+1)y = e^{4t}, \qquad y(1) = 3.$$

9. Consider the differential equation

$$\dot{y} + \frac{\alpha}{t} y = t.$$

   (a) Find the general solution for any α. Be sure to consider the special case where $\alpha = -2$.
   (b) For what values of α does the solution stay bounded as $t \to \infty$?
   (c) Where is your solution discontinuous?

10. In #3, you showed that for the driven RL circuit shown above, the governing equation for the current I is given by

$$\dot{I} + \frac{R}{L} I = \frac{A \cos \omega t}{L}.$$

   (a) If the initial current is zero, $R = 2\,\Omega$, $A = 1$ V, and $L = 2$ H, show that

$$I(t) = \frac{\cos \omega t - e^{-t} + \omega \sin \omega t}{2(\omega^2 + 1)}. \tag{1.5}$$

   Do the initial conditions matter as $t \to \infty$? (You may do complicated integrals using a computer, but you must provide a printout.)

   Often when studying electrical circuits the imposed current or voltage is turned on or off suddenly. In the equation

$$\dot{y} + p(t) y = g(t),$$

   this corresponds to $g(t)$ being discontinuous. When this occurs, we solve the problem in each interval where g is continuous, and then require that y be continuous where the intervals join together.

   Consider the circuit above, but now suppose that we shut the voltage off after one cycle, so we have

$$V = \begin{cases} A \cos \omega t, & 0 \le t \le 2\pi/\omega, \\ 0, & t > 2\pi/\omega. \end{cases}$$

   (b) Using your answer to (a) (with $A = 1$ V), calculate $I(2\pi/\omega)$.