MATH 349-080 Elementary Linear Algebra
Prof. D. A. Edwards Distributed: Dec. 9, 2024

Updates (Revised)

1. This is not a homework assignment that I am collecting, since there isn't enough time left in the term. However, these are the questions I **would** assign if we had time, so they are indicative of the material I feel is important.
2. There will be an informal review session for the final on Friday, Dec. 13 from 1–3 **IN PRN 327**.
3. The final for this class will be administered on Sunday, Dec. 15 from 2:00–4:00 **in GOR 204**. You will need to bring a small blue book, as well as your laptop.
4. Teacher evaluations may also be given at `ratemyprofessors.com` using the QR code at right.

Supplemental Study Material

Read section 4.4.

Section 4.4

1. (BH) Calculate the matrix A which has the following properties:

$$\lambda_1(A) = 2, \qquad \lambda_2(A) = -5, \qquad \mathbf{z}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \qquad \mathbf{z}_2 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

2. Consider the following matrix and vector:

$$A = \begin{pmatrix} -5 & -3 \\ 6 & 4 \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}.$$

   (a) (BH) Find the eigenvalues of A.
   (b) (BH) Find the corresponding eigenvectors of A.
   (c) (BH) Write the spectral decomposition $A = S\Lambda S^{-1}$.
   (d) (BH) Calculate $[A\mathbf{x}]_Z$ and $\Lambda[\mathbf{x}]_Z$. Verify that they are equal.
   (e) (BH) Use your answer to part (c) to calculate $A^{10}\mathbf{x}$.
   (f) (MP) Calculate $A^{10}\mathbf{x}$ and check your answer with (e).

3. Let A and B be similar.
   (a) (BH) Show that if A is nonsingular, then B is nonsingular.
   (b) (BH) If A is nonsingular, then A^{-1} and B^{-1} are similar.

   Now consider the particular matrices

$$A = \begin{pmatrix} 4 & 4 & 2 & 3 & -2 \\ 0 & 1 & -2 & -2 & 2 \\ 6 & 12 & 11 & 2 & -4 \\ 9 & 20 & 10 & 10 & -6 \\ 15 & 28 & 14 & 5 & -3 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

   (Do **NOT** use these matrices for parts (a) and (b); those proofs should be for *any* A and B.)
   (c) (MP) Verify that A and B are similar and invertible, and compute the matrix P such that $A = PBP^{-1}$.
   (d) (MP) Verify that A^{-1} and B^{-1} are similar, and compute the matrix Q such that $A^{-1} = QB^{-1}Q^{-1}$.
   (e) (BH) Show that it is possible to choose $P = Q$. Is that true of the matrices in Mathematica? What does that tell you about the uniqueness of P and Q?