Use of Mathematica

Each problem has a code that indicates how you are to use Mathematica on your submitted work. (You may always use Mathematica to check your work.) The codes are:

BH: (by hand) anything submitted should be done by hand and should show all steps.

MP: (Mathematica printout) use Mathematica as appropriate to get the answer and give me a printout of your worksheet.

Homework Set 1

As I mentioned in class, the material covered this week will be scattered throughout the book. Here are the pages you should read. If you come to something that wasn't covered in class this week, skip it; we will come back to it later on in the course.

Read sections 1.1. 2.3, and 3.5.

Section 1.1

1. Let
$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}, \qquad \mathbf{d} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$

   (a) (MP) Compute the coordinates of $\mathbf{a} + \mathbf{b}$ and $\mathbf{c} - \mathbf{d}$.
   (b) (BH) Sketch the vectors $\{\mathbf{a}, \mathbf{b}, \mathbf{a} + \mathbf{b}\}$ and $\{\mathbf{c}, \mathbf{d}, \mathbf{c} - \mathbf{d}\}$ and compute the coordinates geometrically.

2. (BH) Let
$$\mathbf{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} 4 \\ r \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} s \\ 2 \end{pmatrix}.$$

   Find r and s so that
   (a) $\mathbf{z} = -2\mathbf{x}$.
   (b) $3\mathbf{u}/2 = \mathbf{y}$.
   (c) $\mathbf{z} - \mathbf{u} = \mathbf{x}$.

3. (BH) A box of cereal contains a mixture of b grams of bran flakes and c grams of corn flakes. The specifications of each are as follows:

Property	**bran flakes (per gram)**	**corn flakes (per gram)**
calories	3.2	3.6
fiber (g)	0.17	0.04
Iron (% of daily allowance)	1.5	1.1

   (a) How many calories are in the box?
   (b) We want to express the content of the box as a three-dimensional vector consisting of calories, fiber, and iron. Express this output as a linear combination of two vectors.

Section 2.3

4. (MP)
   (a) Compute a set of three vectors in $\mathcal{R}^3$ with random entries in $[0, 6]$. Is the set linearly independent?
   (b) Redo your computation with three different sets. What do your results lead you to conjecture about linear independence?

5. (BH) exercise 46

6. (BH) Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in a vector space V. Prove that $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is also linearly independent, where $\mathbf{w}_1 = \mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3$, $\mathbf{w}_2 = \mathbf{v}_2 + 2\mathbf{v}_3$, and $\mathbf{w}_3 = 4\mathbf{v}_3$.

7. (BH) Show that

$$\text{Span}\left\{\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}\right\} = \text{Span}\left\{\begin{pmatrix} 2 \\ -5 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 12 \\ -5 \end{pmatrix}\right\}.$$

8. (BH) Consider the following vectors:

$$\mathbf{x}_1 = \begin{pmatrix} -3 \\ 1 \\ 6 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}, \quad \mathbf{x}_3 = \begin{pmatrix} 1 \\ z \\ -2 \end{pmatrix}.$$

   (a) For which value(s) of z does the equation $\mathbf{x}_3 = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$ have a solution?
   (b) For which value(s) of z are $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ linearly dependent?

Section 3.5

9. Consider the following set:

$$W = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -1 \\ -7 \end{pmatrix} \right\}.$$

   (a) (MP) Find a basis for $\operatorname{Span} W$.
   (b) (BH) Find $\dim(\operatorname{Span} W)$.

10. (BH) Find a basis for

$$\operatorname{Span} \left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \right\}.$$