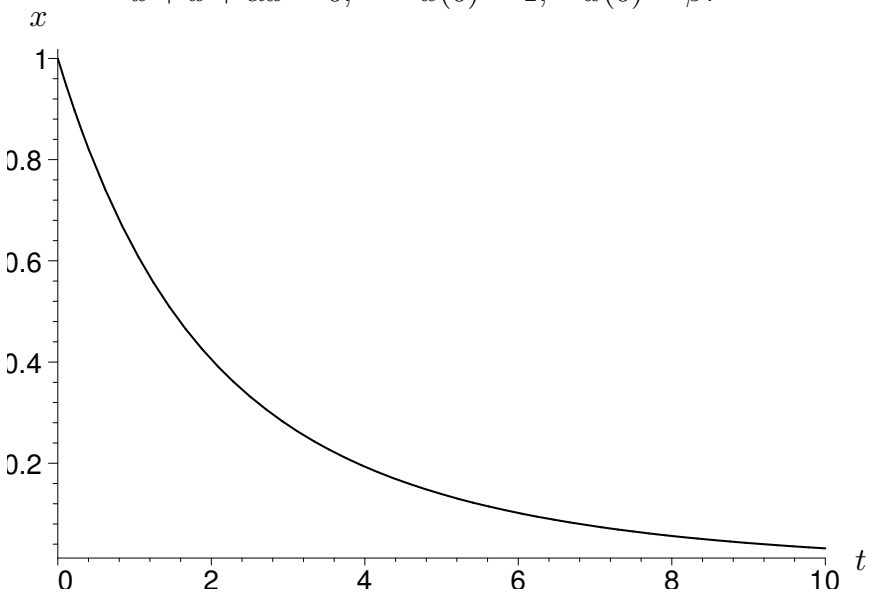
# Unforced Spring System

In class we derived the dimensionless spring system

$$\ddot{x} + \dot{x} + \alpha x = 0, \qquad x(0) = 1, \quad \dot{x}(0) = \beta.$$




$x(t)$ *vs.* $t$ for $\alpha = 0.2$, $\beta = -0.5$.

In this case $\alpha < 1/4$, so the solution is overdamped.

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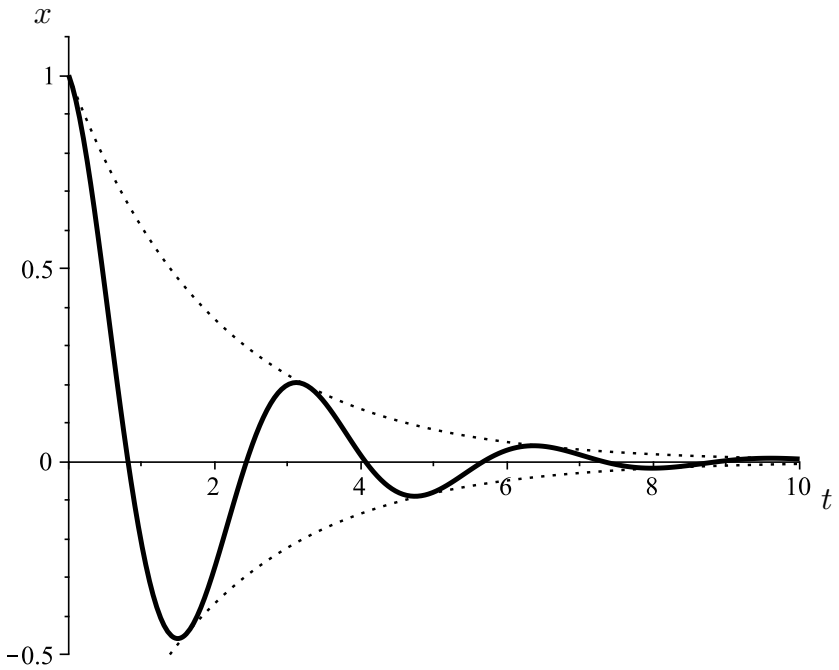
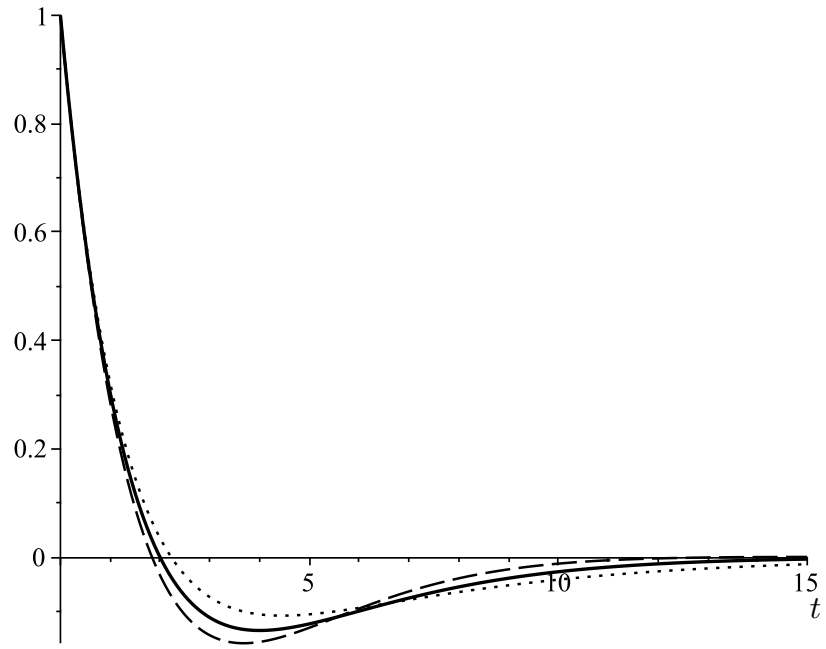



Solid curve: $x(t)$ *vs.* $t$ for $\alpha = 4$, $\beta = -0.5$. Dotted curves: envelopes $\pm e^{-t/2}$.

In this case $\alpha > 1/4$, so the solution is underdamped.




$x(t)$ *vs.* $t$ for $\beta = -1$ and $\alpha = 0.24$ (dotted), 0.25 (solid), and 0.32 (dashed).

Here the solid curve is the critically damped case $\alpha = 1/4$, and the other curves show an underdamped and overdamped case with nearby values of $\alpha$. Note their similarity.