MATH 302-011 Ordinary Differential Equations
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Calculus Review

In calculus, you should have learned that the exponential function is its own derivative. In other words, if $x = e^t$,

$$\dot{x} = \frac{d(e^t)}{dt} = e^t = x.$$

Therefore, by the Chain Rule we have that if $y = e^{\lambda t}$,

$$\dot{y} = \frac{d(e^{\lambda t})}{dt} = \lambda e^{\lambda t} = \lambda y.$$

In MATH 242 we used the method of partial fractions to do integration. In particular, suppose we wanted to integrate

$$\int \frac{dx}{(c_1 x + c_2)(c_3 x + c_4)},$$

where the c_j are all known constants. We rewrite the integrand as

$$\frac{1}{(c_1 x + c_2)(c_3 x + c_4)} = \frac{A}{c_1 x + c_2} + \frac{B}{c_3 x + c_4},$$

since the right-hand side is easy to integrate. To find the unknown constants A and B, we multiply both sides by the denominator of the right-hand side, yielding

$$1 = (\text{coefficients of } x)x + (\text{constants}).$$

Setting the coefficients of x equal to zero and the constants equal to 1 will yield two equations that can be solved for A and B.

We will also be using the following Taylor series this semester:

$$\begin{aligned} e^z &= \sum_{n=0}^{\infty} \frac{z^n}{n!}, \\ \sin z &= \sum_{m=0}^{\infty} \frac{(-1)^m z^{2m+1}}{(2m+1)!}, \\ \cos z &= \sum_{m=0}^{\infty} \frac{(-1)^m z^{2m}}{(2m)!}. \end{aligned}$$