MATH 302-010 Ordinary Differential Equations
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Laplace Transforms

In class, we presented the following pairs:

$$f(t) \quad \Longleftrightarrow \quad \mathcal{L}\{f(t)\} = \hat{f}(s) = \int_0^\infty e^{-st} f(t)\, dt \tag{1}$$

$$e^{at} \quad \Longleftrightarrow \quad \frac{1}{s-a} \tag{3}$$

$$\dot{f} \quad \Longleftrightarrow \quad s\hat{f} - f(0) \tag{4a}$$

$$\ddot{f} \quad \Longleftrightarrow \quad s^2\hat{f} - sf(0) - \dot{f}(0) \tag{4b}$$

$$\sin bt \quad \Longleftrightarrow \quad \frac{b}{s^2+b^2} \tag{6}$$

$$-\,tf(t) \quad \Longleftrightarrow \quad \frac{d\hat{f}}{ds} \tag{8}$$

$$\frac{f(t)}{t} \quad \Longleftrightarrow \quad \int_s^\infty \hat{f}(\sigma)\, d\sigma \tag{9}$$

$$u_c(t)f(t-c) \quad \Longleftrightarrow \quad e^{-cs}\mathcal{L}\{f(t)\} \tag{10}$$

$$e^{ct}f(t) \quad \Longleftrightarrow \quad \hat{f}(s-c) \tag{11}$$

$$\int_0^t f(\tau)g(t-\tau)\, d\tau \quad \Longleftrightarrow \quad \hat{f}(s)\hat{g}(s) \tag{12}$$

(Note that on the right-hand side of (10), the Laplace transform is of the normal form of f, not the shifted form.)

We can also use the properties above to derive additional results. Taking $a = 0$ in (3), we obtain

$$1 \quad \Longleftrightarrow \quad \frac{1}{s}. \tag{2}$$

Substituting (6) into (4a), we obtain

$$\cos bt \quad \Longleftrightarrow \quad \frac{s}{s^2+b^2}. \tag{7}$$

Using (6) in (11), we have

$$e^{at}\sin bt \quad \Longleftrightarrow \quad \frac{b}{(s-a)^2+b^2}. \tag{5}$$