MATH 302-010 — Ordinary Differential Equations

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Inhomogeneous Equations

Suppose that we have the constant-coefficient inhomogeneous ODE

$$a\ddot{y} + b\dot{y} + cy = f(t).$$

Then we may use the *method of undetermined coefficients* to find the particular solution. What to try is given in this table:

Table . 1. Method of Undetermined Coefficients

$f(t)$	Try
$e^{\gamma t}$	$Ae^{\gamma t}$
$\sin\gamma t$ **OR** $\cos\gamma t$	$A_{\mathrm{s}}\sin\gamma t + A_{\mathrm{c}}\cos\gamma t$
t^n(+ other terms)	$a_n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0$
$e^{\gamma t}\sin\beta t$ **OR** $e^{\gamma t}\cos\beta t$	$e^{\gamma t}(A_{\mathrm{s}}\sin\beta t + A_{\mathrm{c}}\cos\beta t)$
$e^{\gamma t}[t^n$(+ other terms)$]$	$e^{\gamma t}(a_n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0)$

The entries in this table work as we are long not forcing at resonance. This method is easier, but doesn't cover every case. It also works for higher-order equations, as long as they have constant coefficients.

Suppose that we have the general second-order inhomogeneous ODE

$$\ddot{y} + p(t)\dot{y} + q(t)y = g(t), \tag{ .1}$$

and you know two solutions y_1 and y_2 of the *homogeneous* equation. Then the particular solution may be found using the *variation of parameters formula*:

$$Y = -y_1\int \frac{y_2(t)g(t)}{W(y_1,y_2)}\,dt + y_2\int \frac{y_1(t)g(t)}{W(y_1,y_2)}\,dt,$$

where W is the Wronskian.

For the formula to work, the equation **MUST** be in the theoretical form (.1). Note this works only for second-order equations.