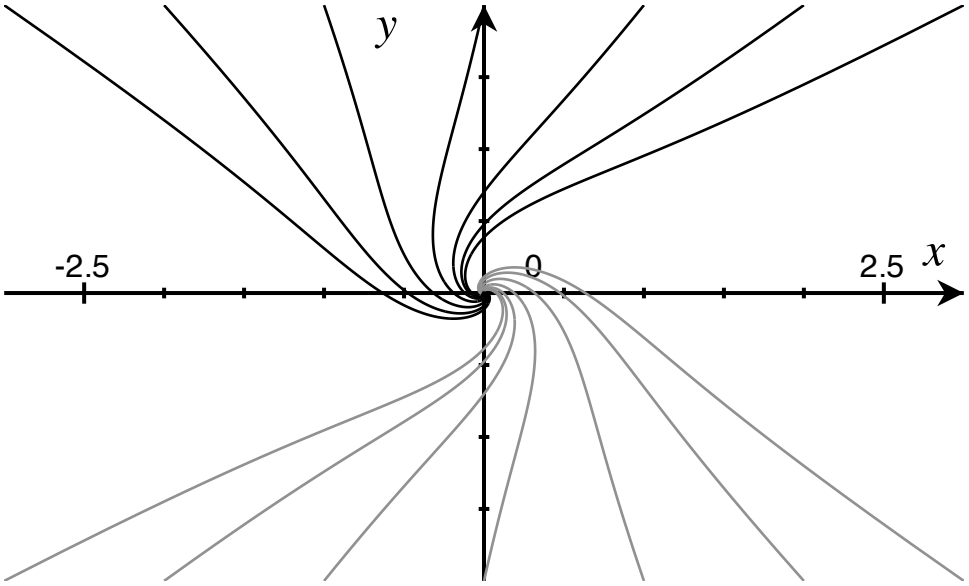
The Hopf Bifurcation

In class, we discussed the solutions of the following system:

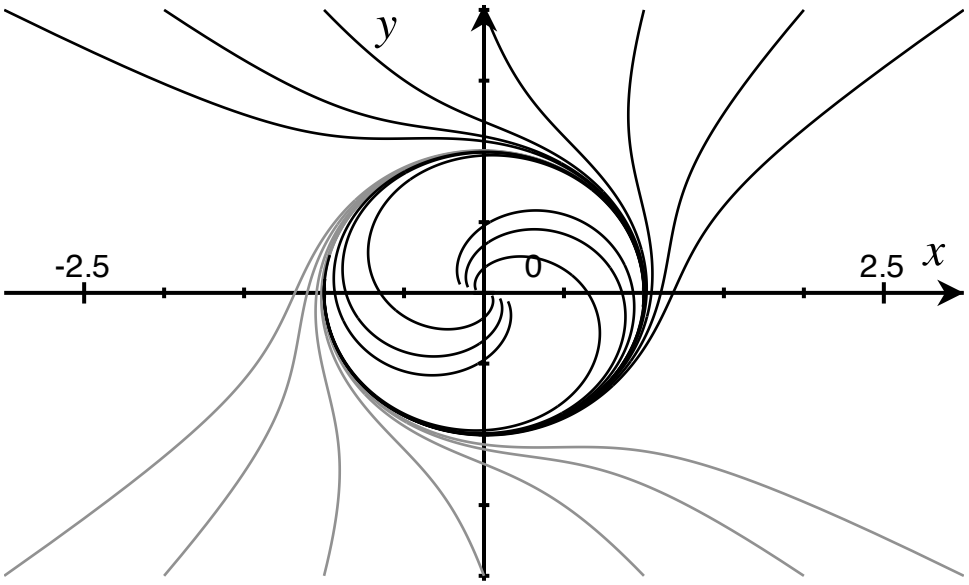
$$\dot{x} = \alpha x - \omega y - x(x^2 + y^2),$$
$$\dot{y} = \omega x + \alpha y - y(x^2 + y^2).$$

We noted that the sign of α determined whether there was a stable limit cycle outside the origin, and the sign of ω determined whether the trajectories spun clockwise or counterclockwise.



$\alpha = -1$, $\omega = 1$. No limit cycle; counterclockwise rotation.

This figure shows the case where α is negative. In this case, the origin is a stable spiral. ω is positive here, so the trajectories spin inward in a counterclockwise direction.



$\alpha = 1$, $\omega = -1$. Limit cycle with clockwise rotation.

This figure shows the case where α is positive. In this case, the origin is an unstable spiral and there is a stable limit cycle at $r = 1$. ω is negative here, so the trajectories spin in a clockwise direction.