# Generalized Eigenvectors

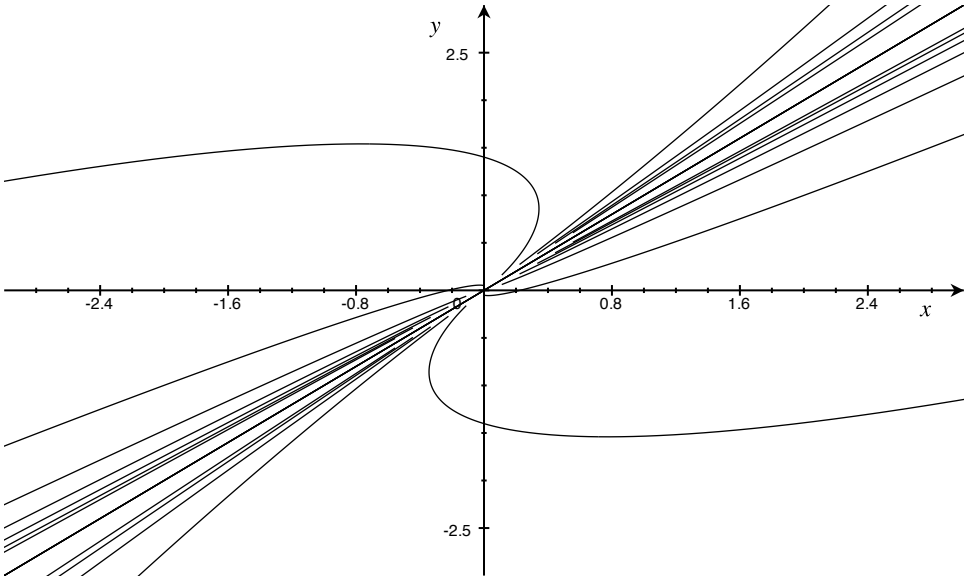
The system

$$\dot{\mathbf{x}} = \begin{pmatrix} 5 & -2 \\ 2 & 1 \end{pmatrix} \mathbf{x} \tag{1}$$

has only one eigenvalue: $\lambda = 3$, and only one eigenvector: $(1,1)^T$. There is a generalized eigenvector $(1/2, 0)^T$, and hence the solution of (1) is given by

$$\mathbf{x} = c_1 e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3t} \left[ t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} \right].$$

The phase plane is shown below. Note that all solutions approach a multiple of $(1,1)^T$.




Phase plane of (1).