Fourier Series

We derived in class that the Fourier series for the function

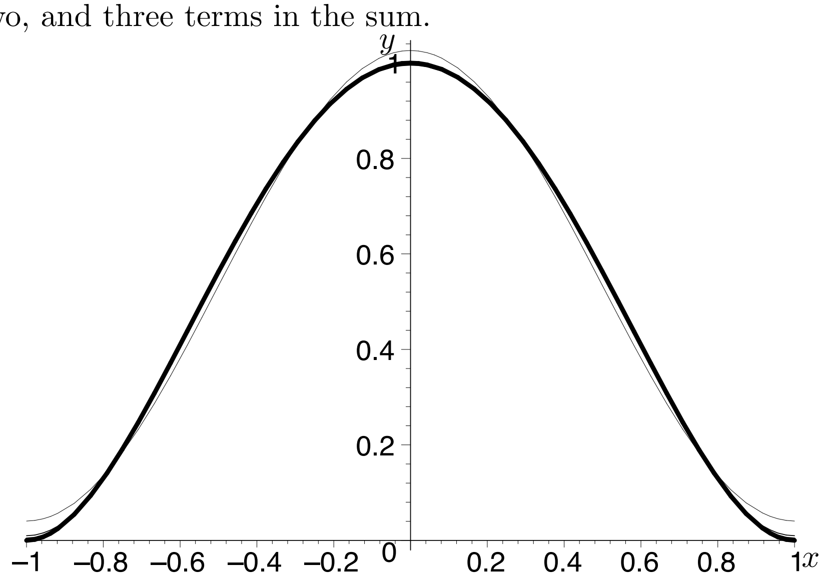
$$f(x) = (x - L)^2(x + L)^2$$

is given by

$$f(x) = \frac{8L^4}{15} - \frac{48L^4}{\pi^4}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}\cos\left(\frac{n\pi x}{L}\right).$$

Note that $f(x)$ is smooth in its domain.

Below are plotted the function f (thickest line), as well as the Fourier series taking the first one, two, and three terms in the sum.



In increasing order of thickness: Fourier series keeping the first one, two, and, three terms in the series, as well as $f(x)$ *vs.* x for $L = 1$.

Remarks

1. Note that with each increasing term, the series becomes a better approximation.
2. If we take the first N terms of the series, the absolute value of the $(N + 1)$st term is a rough estimate of the error. Note this term is always smaller than

$$\frac{48L^4}{(N+1)^4\pi^4},$$

which decays very rapidly as N gets large.

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We derived in class that the Fourier series for the function

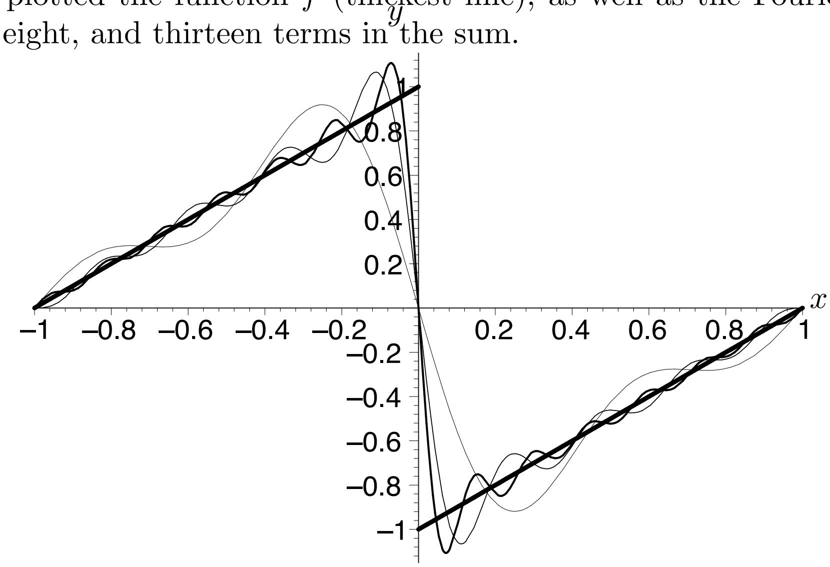
$$f(x) = \begin{cases} x + L, & -L \leq x < 0, \\ x - L, & 0 < x \leq L, \end{cases}$$

is given by

$$f(x) = -\frac{2L}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right).$$

Note that $f(x)$ is *not* smooth in its domain.

Below are plotted the function f (thickest line), as well as the Fourier series taking the first three, eight, and thirteen terms in the sum.



In increasing order of thickness: Fourier series keeping the first three, eight, and, thirteen terms in the series, as well as $f(x)$ *vs.* x for $L = 1$.

Remarks

1. Note that with each increasing term, the series becomes a better approximation throughout most of the domain.
2. If we take the first N terms of the series, the absolute value of the $(N+1)$st term is a rough estimate of the error. Note this term is always smaller than

$$\frac{2L}{(N+1)\pi},$$

which decays very slowly as N gets large.

3. Note that near the discontinuity at $x = 0$ we obtain larger and larger oscillations. This is called the *Gibbs phenomenon*, and arises because we are trying to estimate a discontinuous function by a series of continuous functions. Also note that the approximations all run through the origin, which represents the average of the two discontinuous values.