Homework Set 8 Solutions (11/21 Version)

1. (BH) Use Laplace transforms to find the solution of

$$\ddot{y} + 9y = 1, \qquad y(0) = 0, \quad \dot{y}(0) = 3.$$

Solution. Taking the Laplace transform, we obtain

$$\begin{aligned}
s^2\hat{y} - sy(0) - \dot{y}(0) + 9\hat{y} &= \frac{1}{s} \\
(s^2+9)\hat{y} &= 3 + \frac{1}{s} = \frac{3s+1}{s} \\
\hat{y} &= \frac{3s+1}{s(s^2+9)} = \frac{A}{s} + \frac{Bs}{s^2+9} + \frac{C}{s^2+9} \\
3s+1 &= A(s^2+9) + s(Bs+C).
\end{aligned}$$

Using the trick of substituting $s = 0$ into both sides, we have $1 = 9A$, or $A = 1/9$, yielding

$$3s + 1 = \frac{s^2+9}{9} + Bs^2 + Cs.$$

Matching the coefficients of s^2, we have $B = -1/9$. Similarly, matching the coefficients of s, we have $C = 3$. Hence we have

$$\begin{aligned}
\hat{y} &= \frac{1}{9s} - \frac{s}{9(s^2+9)} + \frac{3}{s^2+9} \\
y(t) &= \frac{1-\cos 3t}{9} + \sin 3t,
\end{aligned}$$

where we have used the tables.

2. Consider the following system of differential equations for the three unknowns $\{x(t), y(t), z(t)\}$:

$$\dot{x} + 2x + z = 4, \tag{8.1a}$$
$$\dot{y} + 2y - z = 0, \tag{8.1b}$$
$$\dot{z} = x - y, \tag{8.1c}$$
$$x(0) = 1, \qquad y(0) = 1, \qquad z(0) = 0. \tag{8.2}$$

(a) (BH) Use Laplace transforms to show that $x(t) + y(t) = 2$.

Solution. Taking the Laplace transform of (8.1a) and (8.1b) subject to (8.2), we obtain

$$\begin{aligned} s\hat{x} - 1 + 2\hat{x} + \hat{z} &= \frac{4}{s} \\ s\hat{y} - 1 + 2\hat{y} - \hat{z} &= 0 \\ (s+2)(\hat{x}+\hat{y}) - 2 &= \frac{4}{s} \end{aligned} \tag{A.1}$$

$$\hat{x} + \hat{y} = \frac{1}{s+2}\left(2 + \frac{4}{s}\right) = \frac{2s+4}{s(s+2)} = \frac{2}{s} \tag{A.2}$$

$$x + y = 2.$$

(b) (BH) Use Laplace transforms to show that

$$\hat{x} = \frac{s^2+4s+2}{s(s^2+2s+2)}, \qquad \hat{y} = \frac{s^2+2}{s(s^2+2s+2)}.$$

Solution. Taking the Laplace transform of (8.1c) subject to (8.2), we obtain

$$s\hat{z} = \hat{x} - \hat{y}.$$

Substituting the above into (A.1), we above

$$\begin{aligned} s\hat{y} - 1 + 2\hat{y} - \hat{z} &= 0 \\ (s+2)\hat{y} - \frac{\hat{x}-\hat{y}}{s} &= 1 \\ (s^2+2s)\hat{y} - \hat{x} + \hat{y} &= s, \\ (s^2+2s+1)\hat{y} - \left(\frac{2}{s} - \hat{y}\right) &= s, \end{aligned}$$

where in the last line we have used (A.2) to solve for $\hat{x}$. Continuing to simplify, we obtain

$$\begin{aligned} (s^2+2s+2)\hat{y} &= s + \frac{2}{s} \\ \hat{y} &= \frac{s^2+2}{s(s^2+2s+2)}, \\ \hat{x} = \frac{2}{s} - \frac{s^2+2}{s(s^2+2s+2)} &= \frac{2(s^2+2s+2) - (s^2+2)}{s(s^2+2s+2)} \\ &= \frac{s^2+4s+2}{s(s^2+2s+2)}, \end{aligned}$$

(c) (MP) Invert your answer to part (b) to obtain real solutions for $x(t)$ and $y(t)$. Verify your answer to (a).

3. (BH) Use the appropriate property to show that

$$\mathcal{L}\{te^t\} = \frac{1}{(s-1)^2}.$$

Solution. From notes in class, we have that

$$\mathcal{L}\{te^t\} = -\frac{d(\mathcal{L}\{e^t\})}{ds} = -\frac{d}{ds}\left(\frac{1}{s-1}\right) = \frac{1}{(s-1)^2},$$

as required.

4. (For this problem, you may use Mathematica to do the partial fraction expansion, but do the rest by hand.) Consider the differential equation

$$\ddot{y} - \omega^2 y = e^t, \quad \omega > 0; \qquad y(0) = \dot{y}(0) = 0.$$

(a) (MI) Use Laplace transforms to solve the problem for all $\omega \neq 1$.

Solution. Taking the Laplace transform, we have

$$\begin{aligned} s^2\hat{y} - (0)s - (0) - \omega^2\hat{y} &= \frac{1}{s-1} \\ \hat{y} &= \frac{1}{(s-1)(s^2-\omega^2)} = \frac{1}{(s+\omega)(s-\omega)(s-1)} \qquad \text{(B)} \\ &= \frac{1}{2\omega(\omega+1)}\frac{1}{s+\omega} + \frac{1}{2\omega(\omega-1)}\frac{1}{s-\omega} + \frac{1}{1-\omega^2}\frac{1}{s-1} \\ y(t) &= \frac{e^{-\omega t}}{2\omega(\omega+1)} + \frac{e^{\omega t}}{2\omega(\omega-1)} + \frac{e^t}{1-\omega^2}, \quad \omega \neq 1. \end{aligned}$$

(b) (MI) Use your answer to #3 to show that

$$y(t) = \frac{te^t - \sinh t}{2}, \quad \omega = 1.$$

Solution. If $\omega = 1$, (B) becomes

$$\hat{y} = \frac{1}{(s+1)(s-1)^2} = \frac{1}{4}\left(\frac{1}{s+1} - \frac{1}{s-1}\right) + \frac{1}{2}\frac{1}{(s-1)^2}.$$

Then using #3 to invert the last term, we have

$$y(t) = \frac{e^{-t} - e^t}{4} + \frac{te^t}{2} = \frac{te^t - \sinh t}{2},$$

as required.

5.

(a) (BH) Show that

$$\sum_{n=0}^{\infty} e^{-snT} = \frac{1}{1-e^{-sT}}. \tag{8.3}$$

Solution. We rewrite the sum in a more recognizable form of the geometric series:

$$\sum_{n=0}^{\infty} e^{-snT} = \sum_{n=0}^{\infty} \left(e^{-sT}\right)^n = \frac{1}{1-e^{-sT}},$$

as required.

(b) (BH) Let f satisfy $f(t+T) = f(t)$ for all $t \geq 0$ and some fixed positive number T; f is said to be *periodic* with period T for $t \geq 0$. Show that

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt.$$

Solution. Since the function is periodic with period T, we break up the integral into pieces of length T:

$$\begin{aligned}\mathcal{L}\{f(t)\} &= \int_0^\infty e^{-st} f(t)\, dt = \int_0^T e^{-st} f(t)\, dt + \int_T^{2T} e^{-st} f(t)\, dt + \cdots \\ &= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} e^{-st} f(t)\, dt.\end{aligned}$$

Shifting variables so all the integrals are over the same interval, we take $t = u + nT$ to obtain

$$\mathcal{L}\{f(t)\} = \sum_{n=0}^{\infty} \int_0^T e^{-s(u+nT)} f(u+nT)\, du.$$

But f is periodic, so $f(u+nT) = f(u)$ for any integer n. Thus we have

$$\mathcal{L}\{f(t)\} = \sum_{n=0}^{\infty} e^{-snT} \int_0^T e^{-su} f(u)\, du = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt,$$

where in the last line we have used (8.3).

6. (BH) Recall that the *greatest integer function* $\lfloor t \rfloor$ is given by the greatest integer less than or equal to t. Thus $\lfloor 2 \rfloor = 2$, $\lfloor 2.9995 \rfloor = 2$, $\lfloor 3 \rfloor = 3$, etc.

(a) Explain why

$$\lfloor t \rfloor = \sum_{n=1}^{\infty} u_n(t).$$

Solution. For $t \in [0, 1)$, the function is zero, and then it changes to 1 at $t = 1$. This part acts just like $u_1(t)$. At $t = 2$, it jumps up 1 again, which is equivalent to adding $u_2(t)$.

Similarly, at each integer n, the function jumps up 1, which is equivalent to adding $u_n(t)$. Thus we have that the function is the sum of all such functions $u_n(t)$, so we have

$$\lfloor t \rfloor = \sum_{n=1}^{\infty} u_n(t).$$

(b) Show that

$$\mathcal{L}\{\lfloor t \rfloor\} = \frac{1}{s(e^s - 1)}.$$

Solution. Taking the transform and factoring out e^{-s} so we have a sum from $n = 0$, we have

$$\mathcal{L}\{\lfloor t \rfloor\} = \sum_{n=1}^{\infty} \mathcal{L}\{u_n(t)\} = \sum_{n=1}^{\infty} \frac{e^{-ns}}{s} = \frac{e^{-s}}{s} \sum_{n=0}^{\infty} \left(e^{-s}\right)^n.$$

Then using (8.3) with $T = 1$, we have

$$\mathcal{L}\{\lfloor t \rfloor\} = \frac{e^{-s}}{s} \frac{1}{1 - e^{-s}} = \frac{1}{s(e^s - 1)}.$$

7. Consider the function

$$g(t) = (t-2)^2 u_1(t) - (t-3)^2 u_4(t).$$

(a) (MP) Plot $g(t)$ for $t \in [0, 6]$.
(b) (BH) Calculate $\hat{g}$.

Solution. We note that

$$\begin{aligned} (t-2)^2 u_1(t) &= [(t-1) - 1]^2 u_1(t), \\ (t-3)^2 u_4(t) &= [(t-4) + 1]^2 u_4(t). \end{aligned}$$

Therefore, we have

$$\begin{aligned} \hat{g} &= e^{-s}\mathcal{L}\{(t-1)^2\} - e^{-4s}\mathcal{L}\{(t+1)^2\} \\ &= e^{-s}\mathcal{L}\{t^2 - 2t + 1\} - e^{-4s}\mathcal{L}\{t^2 + 2t + 1\} \\ &= e^{-s}\left(\frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}\right) - e^{-4s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right). \end{aligned}$$

(c) (BH) Find the inverse Laplace transform of

$$\frac{e^{-s}(s-2)}{s^2 + 4}.$$

Solution. The e^{-s} term indicates a shift, so we focus on the rest of the fraction:

$$\mathcal{L}^{-1}\left\{\frac{s-2}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+2^2} - \frac{2}{s^2+2^2}\right\} = \cos 2t - \sin 2t$$
$$\mathcal{L}^{-1}\left\{\frac{e^{-s}(s-2)}{s^2+4}\right\} = u_1(t)[\cos 2(t-1) - \sin 2(t-1)].$$

(d) (BH) Find the inverse Laplace transform of

$$\frac{e^{-(s+2)}}{s^2-4}.$$

Solution. There are several ways to proceed. Here we rewrite the fraction and proceed as in part (c):

$$\frac{e^{-(s+2)}}{s^2-4} = e^{-2}\left[\frac{e^{-s}}{(s+2)(s-2)}\right] = e^{-2}\left[-\frac{e^{-s}}{4}\left(\frac{1}{s+2} - \frac{1}{s-2}\right)\right]$$
$$\mathcal{L}^{-1}\left\{\frac{1}{s+2} - \frac{1}{s-2}\right\} = e^{-2t} - e^{2t}$$
$$\mathcal{L}^{-1}\left\{\frac{e^{-(s+2)}}{s^2-4}\right\} = -\frac{e^{-2}u_1(t)}{4}[e^{-2(t-1)} - e^{2(t-1)}] = \frac{u_1(t)[e^{2(t-2)} - e^{-2t}]}{4}.$$

8. (BH) Find the solution of

$$\ddot{w} - \dot{w} - 2w = 2 - u_2(t), \qquad w(0) = 1, \quad \dot{w}(0) = -1.$$

Solution. Taking the Laplace transform of the above, we obtain

$$s^2\hat{w} - s + 1 - (s\hat{w} - 1) - 2\hat{w} = \frac{2 - e^{-2s}}{s}$$
$$(s^2 - s - 2)\hat{w} = s - 2 + \frac{2 - e^{-2s}}{s}$$
$$\hat{w} = \frac{1}{(s+1)(s-2)}\left(s - 2 + \frac{2 - e^{-2s}}{s}\right) = \frac{1}{s+1} + \frac{2 - e^{-2s}}{s(s-2)(s+1)}.$$

We ignore the exponential when doing the partial fractions, so we have

$$\frac{1}{s(s-2)(s+1)} = \frac{c_1}{s} + \frac{c_2}{s-2} + \frac{c_3}{s+1}$$
$$1 = c_1(s-2)(s+1) + c_2 s(s+1) + c_3 s(s-2). \tag{C}$$

Using our trick, we substitute $s = 0$ into (C) to obtain $1 = -2c_1$, so $c_1 = -1/2$. Then substituting $s = 2$ into (C), we have $1 = 6c_2$, so $c_2 = 1/6$. Lastly, substituting $s = -1$ into (C), we have $1 = 3c_3$, so $c_3 = 1/3$:

$$\begin{aligned}\hat{w} &= \frac{1}{s+1} + (2 - e^{-2s})\left[-\frac{1}{2s} + \frac{1}{6(s-2)} + \frac{1}{3(s+1)}\right]\\ w(t) &= e^{-t} + 2\left(-\frac{1}{2} + \frac{e^{2t}}{6} + \frac{e^{-t}}{3}\right) - u_2(t)\left[-\frac{1}{2} + \frac{e^{2(t-2)}}{6} + \frac{e^{-(t-2)}}{3}\right]\\ &= \frac{5e^{-t}}{3} - 1 + \frac{e^{2t}}{3} - u_2(t)\left[-\frac{1}{2} + \frac{e^{2(t-2)}}{6} + \frac{e^{-(t-2)}}{3}\right].\end{aligned}$$

9. Consider the problem

$$\ddot{v} + \pi^2 v = f(t), \quad v(0) = 1, \quad \dot{v}(0) = 0; \qquad f(t) = -u_0(t) - 2\sum_{k=1}^{n}(-1)^k u_k(t). \tag{8.4}$$

(a) (MP) Plot $f(t)$ for $n = 15$, $t \in [0, 40]$.
(b) (MP) Plot your solution $v(t)$ for $n = 10$, 15, and 20, and $t \in [0, 40]$.
(c) (BH) What do you think happens as $n \to \infty$?

Solution. In each subsequent graph, the amplitude oscillations are growing for a longer period of time and are thus growing larger. Thus as $n \to \infty$, we expect continued growth for an infinite period of time, leading to infinitely large oscillations.

10. In Homework Set 5, #5, we considered the following problem:

$$\ddot{x} + \frac{1}{32}\dot{x} + 96x = F(t), \qquad x(0) = 1/7, \quad \dot{x}(0) = 0, \tag{8.5}$$

where

$$F(t) = \begin{cases} 4\sin t, & 0 \le t \le 2\pi, \\ 0, & t > 2\pi. \end{cases}$$

(a) (BH) Write $F(t)$ in unit-step notation.

Solution. Since $F(t)$ turns off at $t = 2\pi$, we have

$$F(t) = [1 - u_{2\pi}(t)](4\sin t) = 4[\sin t - u_{2\pi}(t)\sin(t - 2\pi)],$$

since $\sin t$ is periodic.

(b) (BH) Show that

$$\hat{x} = \left(s^2 + \frac{s}{32} + 96\right)^{-1}\left[4\left(\frac{1 - e^{-2\pi s}}{s^2 + 1}\right) + \frac{1}{7}\left(s + \frac{1}{32}\right)\right]. \tag{8.6}$$

Solution. Taking the Laplace transform of (8.5) using our answer to (a), we have

$$\begin{aligned} s^2\hat{x} - s\left(\frac{1}{7}\right) - 0 + \frac{1}{32}\left(s\hat{x} - \frac{1}{7}\right) + 96\hat{x} &= 4\left(\frac{1}{s^2+1} - \frac{e^{-2\pi s}}{s^2+1}\right) \\ \hat{x}\left(s^2 + \frac{s}{32} + 96\right) &= 4\left(\frac{1-e^{-2\pi s}}{s^2+1}\right) + \frac{1}{7}\left(s + \frac{1}{32}\right) \end{aligned}$$

$$\hat{x} = \left(s^2 + \frac{s}{32} + 96\right)^{-1}\left[4\left(\frac{1-e^{-2\pi s}}{s^2+1}\right) + \frac{1}{7}\left(s + \frac{1}{32}\right)\right],$$

as required.

(c) (MP) Invert (8.6) to determine the solution for $x(t)$.

(d) (MP) Plot your solution x for $t \in [0, 4\pi]$.

In[ ]:= Quit[]

HW1 (Checked)

HW2 (Checked)

HW3 (Checked)

HW4 (Checked)

HW5 (Checked)

HW6 (Checked)

HW7 (Checked)

HW8 (Revised 11/19)

Number 2c.

Note that we have to convert our exponential solutions to trigonometric functions.

```
sol2cx = (s^2 + 4 * s + 2) / s / (s^2 + 2 * s + 2)
sol2cy = (s^2 + 2) / s / (s^2 + 2 * s + 2)
InverseLaplaceTransform[sol2cx, s, t]
xsol = Simplify[ExpToTrig[%]]
InverseLaplaceTransform[sol2cy, s, t]
ysol = Simplify[ExpToTrig[%]]
```

Out[ ]=

$$\frac{2+4\,s+s^2}{s\,(2+2\,s+s^2)}$$

Out[ ]=

$$\frac{2+s^2}{s\,(2+2\,s+s^2)}$$

Out[ ]=

$$1-i\,e^{(-1-i)\,t}\,(-1+e^{2\,i\,t})$$

Out[ ]=

$$1+e^{-t}\,\mathrm{Sin}[t]-e^{-t}\,\mathrm{Cos}[2\,t]\,\mathrm{Sin}[t]+e^{-t}\,\mathrm{Cos}[t]\,\mathrm{Sin}[2\,t]+ i\,(e^{-t}\,\mathrm{Cos}[t]-e^{-t}\,\mathrm{Cos}[t]\,\mathrm{Cos}[2\,t]-e^{-t}\,\mathrm{Sin}[t]\,\mathrm{Sin}[2\,t])$$

Out[ ]=

$$1+2\,\mathrm{Cosh}[t]\,\mathrm{Sin}[t]-2\,\mathrm{Sin}[t]\,\mathrm{Sinh}[t]$$

Out[ ]=

$$1+i\,e^{(-1-i)\,t}\,(-1+e^{2\,i\,t})$$

Out[ ]=

$$1-2\,\mathrm{Cosh}[t]\,\mathrm{Sin}[t]+2\,\mathrm{Sin}[t]\,\mathrm{Sinh}[t]$$

These answers sum to 2, as expected:

In[ ]:=

```
xsol + ysol
```

Out[ ]=

2

Number 4a.

First we transform the equation; then we substitute in the initial conditions. Once we have done that, we solve for the Laplace transform and do the partial fraction expansion.

In[∘]:=
```
eq4 = y''[t] - omega^2 * y[t] == Exp[t]
ic4 = {y[0] → 0, y'[0] → 0}
LaplaceTransform[eq4, t, s]
% /. ic4
Solve[%, LaplaceTransform[y[t], t, s]]
Apart[%, s]
```

Out[∘]=

$$-\text{omega}^2\, y[t] + y''[t] == e^t$$

Out[∘]=

{y[0] → 0, y′[0] → 0}

Out[∘]=

$$-\text{omega}^2\, \text{LaplaceTransform}[y[t], t, s] + s^2\, \text{LaplaceTransform}[y[t], t, s] - s\, y[0] - y'[0] == \frac{1}{-1+s}$$

Out[∘]=

$$-\text{omega}^2\, \text{LaplaceTransform}[y[t], t, s] + s^2\, \text{LaplaceTransform}[y[t], t, s] == \frac{1}{-1+s}$$

Out[∘]=

$$\left\{\left\{\text{LaplaceTransform}[y[t], t, s] \to \frac{1}{(-1+s)\left(-\text{omega}^2+s^2\right)}\right\}\right\}$$

Out[∘]=

$$\left\{\left\{\text{LaplaceTransform}[y[t], t, s] \to -\frac{1}{(-1+\text{omega})(1+\text{omega})(-1+s)} + \frac{1}{2(-1+\text{omega})\,\text{omega}\,(-\text{omega}+s)} + \frac{1}{2\,\text{omega}\,(1+\text{omega})(\text{omega}+s)}\right\}\right\}$$

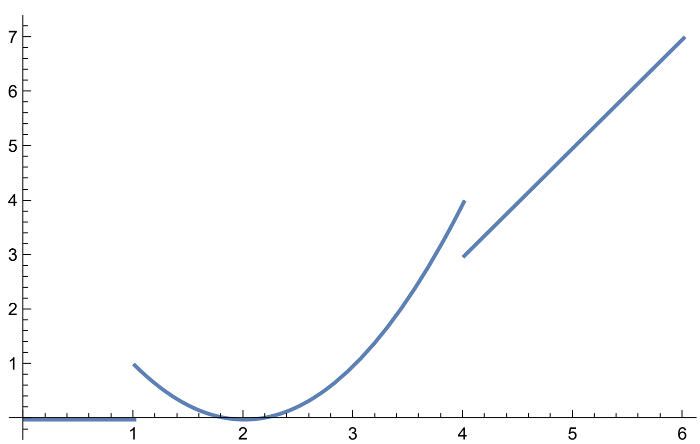
Number 7a.

In[∘]:=
```
g3 = (t - 2) ^2 * HeavisideTheta[t - 1] - (t - 3) ^2 * HeavisideTheta[t - 4]
Plot[g3, {t, 0, 6}]
```

Out[∘]=

$$-(-3+t)^2\, \text{HeavisideTheta}[-4+t] + (-2+t)^2\, \text{HeavisideTheta}[-1+t]$$

Out[∘]=



Number 9a.

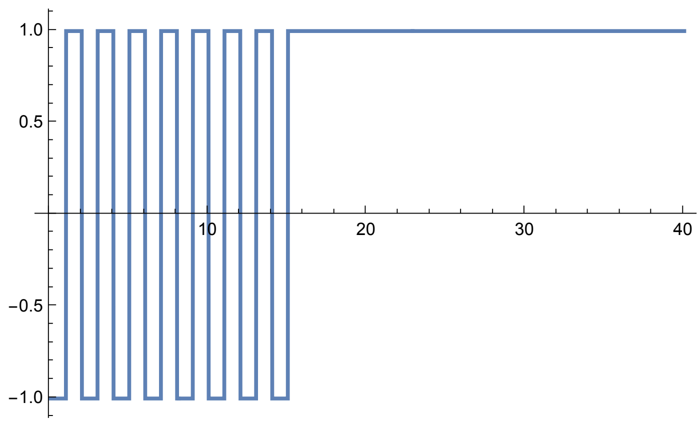
Here there are two (equivalent) graphs, depending on how you define the function:

In[ ]:=
```
f5 = -HeavisideTheta[t] - 2 * Sum[(-1) ^ k * HeavisideTheta[t - k], {k, 1, n}]
Plot[f5 /. (n → 15), {t, 0, 40}]
```

Out[ ]=

$$-\text{HeavisideTheta}[t] - 2 \left(\begin{cases} -1 & (n == 1 \,\&\&\, t \geq 1) \,||\, (n > 1 \,\&\&\, t == 1) \\ \frac{1}{2}\left(-1 + (-1)^{n}\right) & n > 1 \,\&\&\, n - t < 0 \\ \frac{1}{2}\left(-1 + (-1)^{\text{Floor}[t]}\right) & n > 1 \,\&\&\, t > 1 \,\&\&\, n - t \geq 0 \\ 0 & \text{True} \end{cases} \right)$$

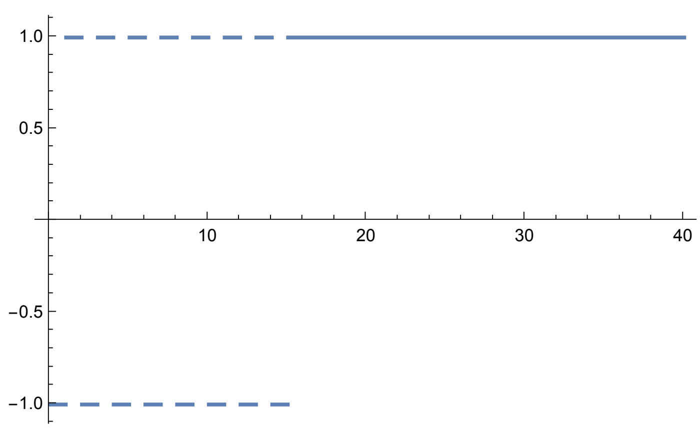
Out[ ]=



In[1]:=
```
f5a = -HeavisideTheta[t] - 2 * Sum[(-1) ^ k * HeavisideTheta[t - k], {k, 1, 15}]
Plot[f5a, {t, 0, 40}]
```

Out[1]= -2 (-HeavisideTheta[-15 + t] + HeavisideTheta[-14 + t] - HeavisideTheta[-13 + t] + HeavisideTheta[-12 + t] - HeavisideTheta[-11 + t] + HeavisideTheta[-10 + t] - HeavisideTheta[-9 + t] + HeavisideTheta[-8 + t] - HeavisideTheta[-7 + t] + HeavisideTheta[-6 + t] - HeavisideTheta[-5 + t] + HeavisideTheta[-4 + t] - HeavisideTheta[-3 + t] + HeavisideTheta[-2 + t] - HeavisideTheta[-1 + t]) - HeavisideTheta[t]

Out[2]=

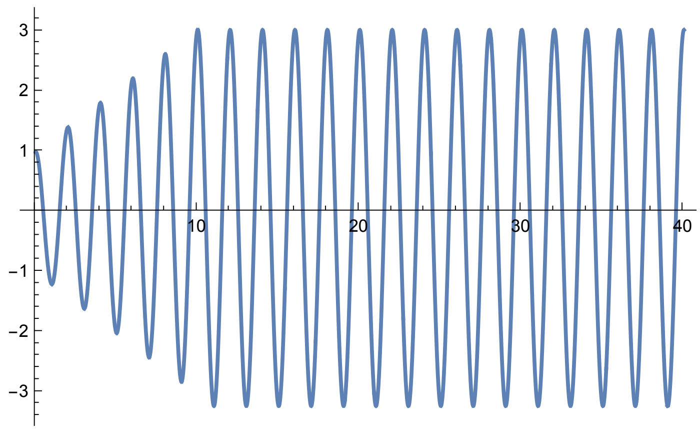


Number 9b.

Here we use the semicolons to hide the lengthy forms of our expressions.

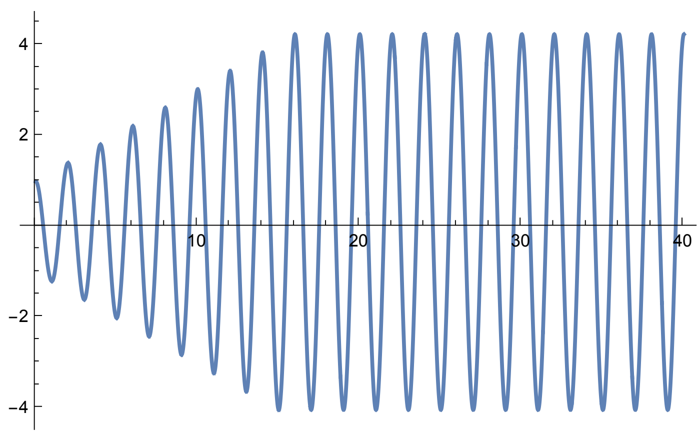
In[ ]:=
```
eq510 = v''[t] + Pi^2 * v[t] == f5 /. (n → 10);
DSolve[{eq510, v[0] == 1, v'[0] == 0}, v[t], t];
sol510 = %[[1, 1, 2]];
Plot[sol510, {t, 0, 40}]
```

Out[ ]=



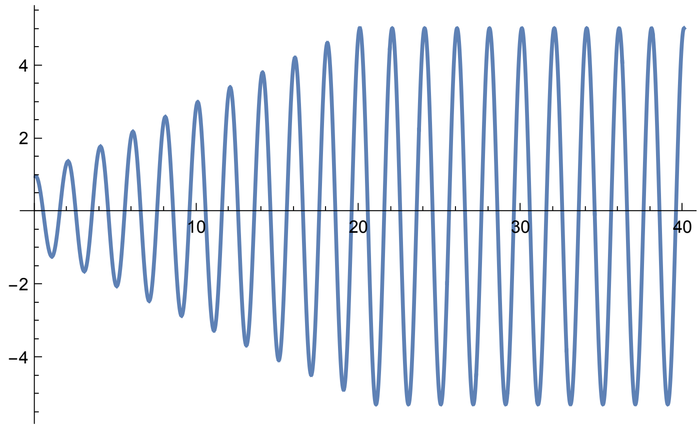
In[ ]:=
```
eq515 = v''[t] + Pi^2 * v[t] == f5 /. (n → 15);
DSolve[{eq515, v[0] == 1, v'[0] == 0}, v[t], t];
sol515 = %[[1, 1, 2]];
Plot[sol515, {t, 0, 40}]
```

Out[ ]=



In[ ]:=
```
eq520 = v''[t] + Pi^2 * v[t] == f5 /. (n → 20);
DSolve[{eq520, v[0] == 1, v'[0] == 0}, v[t], t];
sol520 = %[[1, 1, 2]];
Plot[sol520, {t, 0, 40}]
```

Out[ ]=



Number 10c.

In[ ]:=
```
lap10 = (4 * (1 - Exp[-2 * Pi * s]) / (s^2 + 1) + 1 / 7 * (s + 1 / 32)) / (s^2 + s / 32 + 96)
sol10 = InverseLaplaceTransform[lap10, s, t]
```

Out[ ]=

$$\frac{\frac{1}{7}\left(\frac{1}{32}+s\right)+\frac{4\left(1-e^{-2\pi s}\right)}{1+s^2}}{96+\frac{s}{32}+s^2}$$

Out[ ]=

$$4\left(-\frac{32\,(\text{Cos}[t]-3040\,\text{Sin}[t])}{9\,241\,601}+\frac{32\,e^{-t/64}\left(\sqrt{393\,215}\,\text{Cos}\left[\frac{\sqrt{393\,215}\,t}{64}\right]-194\,559\,\text{Sin}\left[\frac{\sqrt{393\,215}\,t}{64}\right]\right)}{9\,241\,601\,\sqrt{393\,215}}\right)+$$

$$\frac{e^{-t/64}\left(\sqrt{393\,215}\,\text{Cos}\left[\frac{\sqrt{393\,215}\,t}{64}\right]-\text{Sin}\left[\frac{\sqrt{393\,215}\,t}{64}\right]\right)}{7\,\sqrt{393\,215}}+\frac{2\,e^{-t/64}\,\text{Sin}\left[\frac{\sqrt{393\,215}\,t}{64}\right]}{7\,\sqrt{393\,215}}-$$

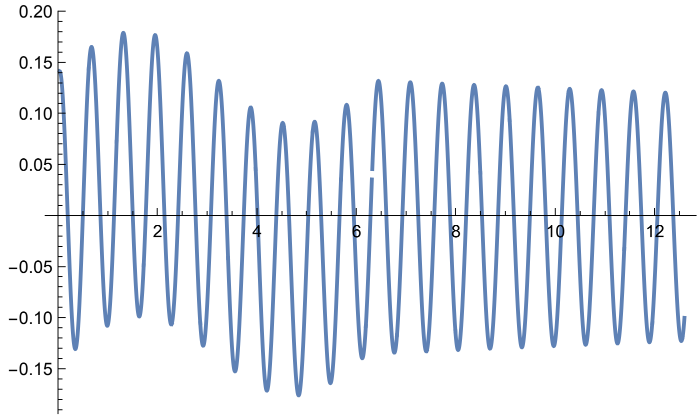
$$4\,\text{HeavisideTheta}[-2\pi+t]\left(-\frac{32\,(\text{Cos}[t]-3040\,\text{Sin}[t])}{9\,241\,601}+\frac{1}{9\,241\,601\,\sqrt{393\,215}}32\,e^{\frac{1}{64}(2\pi-t)}\right.$$

$$\left.\left(\sqrt{393\,215}\,\text{Cos}\left[\frac{1}{64}\sqrt{393\,215}\,(-2\pi+t)\right]-194\,559\,\text{Sin}\left[\frac{1}{64}\sqrt{393\,215}\,(-2\pi+t)\right]\right)\right)$$

Number 10d.

In[ ]:=
```
Plot[sol10, {t, 0, 4*Pi}]
```

Out[ ]=



HW9 (Checked)

SSM (Checked)