MATH 302-010 Ordinary Differential Equations
Prof. D. A. Edwards Due: Oct. 25, 2024

Homework Set 6 Solutions

1. (BH) Consider the following matrix and vectors:

$$A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}, \qquad \mathbf{v}_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix} -2 \\ -4 \end{pmatrix}.$$

   (a) Show by direct multiplication that $\mathbf{v}_1$ and $\mathbf{v}_2$ are eigenvectors of A, and find the corresponding eigenvalues.

Solution.

$$A\mathbf{v}_1 = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 3 \\ 3 \end{pmatrix} \qquad \Longrightarrow \qquad \lambda_1 = 2,$$
$$A\mathbf{v}_2 = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ -12 \end{pmatrix} = 3\begin{pmatrix} -2 \\ -4 \end{pmatrix} \qquad \Longrightarrow \qquad \lambda_2 = 3.$$

   (b) Consider the three vectors $-\mathbf{v}_1$, $3\mathbf{v}_2$, and $-\mathbf{v}_1 + 2\mathbf{v}_2$. Determine by direct multiplication which (if any) are eigenvectors, and find the corresponding eigenvalues.

Solution.

$$A(-\mathbf{v}_1) = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ -3 \end{pmatrix} = \begin{pmatrix} -6 \\ -6 \end{pmatrix} = 2\begin{pmatrix} -3 \\ -3 \end{pmatrix} \qquad \Longrightarrow \qquad \lambda = 2,$$
$$A(3\mathbf{v}_2) = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} -6 \\ -12 \end{pmatrix} = \begin{pmatrix} -18 \\ -36 \end{pmatrix} = 3\begin{pmatrix} -6 \\ -12 \end{pmatrix} \qquad \Longrightarrow \qquad \lambda = 3,$$
$$A(-\mathbf{v}_1 + 3\mathbf{v}_2) = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} -9 \\ -15 \end{pmatrix} = \begin{pmatrix} -24 \\ -42 \end{pmatrix}.$$

$A(-\mathbf{v}_1 + 3\mathbf{v}_2)$ is not a multiple of $-\mathbf{v}_1 + 3\mathbf{v}_2$, so $-\mathbf{v}_1 + 3\mathbf{v}_2$ is not an eigenvector for A.

2. Consider the following matrix and vector function:

$$B = \begin{pmatrix} -3 & 6 \\ 1 & -2 \end{pmatrix}, \qquad \mathbf{w}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{-5t}, \qquad \mathbf{w}_2 = t\mathbf{w}_1.$$

   (a) (BH) Show by direct multiplication that $\dot{\mathbf{w}}_1 = B\mathbf{w}_1$.

Solution.

$$\dot{\mathbf{w}}_1 = -5\begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{-5t} = \begin{pmatrix} 15 \\ -5 \end{pmatrix} e^{-5t}$$
$$B\mathbf{w}_1 = \begin{pmatrix} -3 & 6 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{-5t} = \begin{pmatrix} 15 \\ -5 \end{pmatrix} e^{-5t}$$

(b) (MP) Show by direct multiplication that $\dot{\mathbf{w}}_2 = B\mathbf{w}_2 + \mathbf{w}_1$.

3. Consider the following matrix and vector:

$$C = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}.$$

(a) (BH) Calculate $\det C$.

Solution.

$$\det C = \begin{vmatrix} 5 & 4 \\ 1 & 2 \end{vmatrix} = (5)(2) - (4)(1) = 6.$$

(b) (BH) Calculate C^{-1}.

Solution. Using the inverse formula for 2×2 matrices, we have

$$C^{-1} = \frac{1}{\det C} \begin{pmatrix} 2 & -4 \\ -1 & 5 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 2 & -4 \\ -1 & 5 \end{pmatrix}.$$

(c) (BH) Solve $C\mathbf{x} = \mathbf{b}$.

Solution. The solution is given by

$$\mathbf{x} = C^{-1}\mathbf{b} = \frac{1}{6} \begin{pmatrix} 2 & -4 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 7 \\ -1 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 18 \\ -12 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

(d) (MP) Check your answers to (a)–(c) with Mathematica.

4. (BH) Prove that $\lambda = 0$ is an eigenvalue of A if and only if A is singular.

Solution. We must prove the statement in both directions. If A is singular, then there is a nonzero vector $\mathbf{z}$ such that $A\mathbf{z} = \mathbf{0} = 0\mathbf{z}$. Thus $\lambda = 0$ is an eigenvalue for A. In the opposite direction, we see that if $\lambda = 0$ is an eigenvalue for A, there is a nonzero vector $\mathbf{z}$ such that $A\mathbf{z} = 0\mathbf{z} = \mathbf{0}$, so A is singular.

5. Consider the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}.$$

(a) (BH) Calculate the characteristic polynomial of A.

Solution.

$$P_A(\lambda) = \begin{vmatrix} 1-\lambda & 3 \\ 4 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 12 = \lambda^2 - 3\lambda - 10.$$

(b) (BH) Find the eigenvalues of A.

Solution. Setting the characteristic polynomial equal to zero, we have

$$\lambda^2 - 3\lambda - 10 = (\lambda - 5)(\lambda + 2) = 0,$$

so $\lambda_1 = 5$, $\lambda_2 = -2$.

(c) (BH) Find the eigenvectors of A.

Solution. Solving for the first eigenvector, we obtain

$$(A - 5I)\mathbf{z}_1 = \begin{pmatrix} -4 & 3 \\ 4 & -3 \end{pmatrix} \mathbf{z}_1 = \mathbf{0}.$$

The first row is minus the first, so we have the single equation $4x - 3y = 0$. For simplicity, we take $y = 4$, which means that $x = 3$. Hence the eigenvector is of the form

$$\mathbf{z}_1 = c_1 \begin{pmatrix} 3 \\ 4 \end{pmatrix},$$

for some constant c_1. Solving for the second eigenvector, we obtain

$$(A + 2I)\mathbf{z}_1 = \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix} \mathbf{z}_1 = \mathbf{0}.$$

The rows are multiples of each other, so we have the single equation $4x + 4y = 0$. For simplicity, we take $x = 1$, which means that $y = -1$. Hence the eigenvector is of the form

$$\mathbf{z}_2 = c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

for some constant c_2.

(d) (MP) Check your answers to (b) and (c) with Mathematica.

6. (BH) Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be solutions of

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{A}$$

and let W be their Wronskian.

(a) Show that

$$\frac{dW}{dt} = \begin{vmatrix} \dot{x}_1^{(1)} & \dot{x}_1^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{vmatrix} + \begin{vmatrix} x_1^{(1)} & x_1^{(2)} \\ \dot{x}_2^{(1)} & \dot{x}_2^{(2)} \end{vmatrix}.$$

Solution.

$$\begin{aligned} \frac{dW}{dt} &= \frac{d}{dt}\begin{vmatrix} x_1^{(1)} & x_1^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{vmatrix} = \frac{d}{dt}\left[x_1^{(1)}x_2^{(2)} - x_1^{(2)}x_2^{(1)}\right] \\ &= \left[\dot{x}_1^{(1)}x_2^{(2)} - \dot{x}_1^{(2)}x_2^{(1)}\right] + \left[x_1^{(1)}\dot{x}_2^{(2)} - x_1^{(2)}\dot{x}_2^{(1)}\right] = \begin{vmatrix} \dot{x}_1^{(1)} & \dot{x}_1^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{vmatrix} + \begin{vmatrix} x_1^{(1)} & x_1^{(2)} \\ \dot{x}_2^{(1)} & \dot{x}_2^{(2)} \end{vmatrix}. \end{aligned}$$

(b) Show that

$$\frac{dW}{dt} = (p_{11} + p_{22})W. \tag{B}$$

Solution. Using (A), we have

$$\begin{aligned}
\begin{vmatrix} \dot{x}_1^{(1)} & \dot{x}_1^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{vmatrix} &= \begin{vmatrix} p_{11}x_1^{(1)} + p_{12}x_2^{(1)} & p_{11}x_1^{(2)} + p_{12}x_2^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{vmatrix} \\
&= \left[p_{11}x_1^{(1)} + p_{12}x_2^{(1)}\right] x_2^{(2)} - \left[p_{11}x_1^{(2)} + p_{12}x_2^{(2)}\right] x_2^{(1)} \\
&= p_{11}x_1^{(1)}x_2^{(2)} - p_{11}x_1^{(2)}x_2^{(1)} = p_{11}W. \\
\begin{vmatrix} x_1^{(1)} & x_1^{(2)} \\ \dot{x}_2^{(1)} & \dot{x}_2^{(2)} \end{vmatrix} &= \begin{vmatrix} x_1^{(1)} & x_1^{(2)} \\ p_{21}x_1^{(1)} + p_{22}x_2^{(1)} & p_{21}x_1^{(2)} + p_{22}x_2^{(2)} \end{vmatrix} \\
&= x_1^{(1)} \left[p_{21}x_1^{(2)} + p_{22}x_2^{(2)}\right] - x_1^{(2)} \left[p_{21}x_1^{(1)} + p_{22}x_2^{(1)}\right] \\
&= p_{22}x_1^{(1)}x_2^{(2)} - p_{22}x_1^{(2)}x_2^{(1)} = p_{22}W \\
\frac{dW}{dt} &= (p_{11} + p_{22})W.
\end{aligned}$$

(c) Solve (B) and show that either W is identically zero or never vanishes.

Solution. Solving (B), we have

$$\begin{aligned}
\frac{dW}{W} &= p_{11}(t) + p_{22}(t) \\
\log W &= \int p_{11}(t) + p_{22}(t)\, dt + A \\
W &= C \exp\left(\int p_{11}(t) + p_{22}(t)\, dt\right),
\end{aligned}$$

where $C = e^A$ is a constant. If $C = 0$, W is identically zero. If $C \neq 0$, W never vanishes.

7. (BH) Consider the vectors

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} 6 \\ t \end{pmatrix}, \qquad \mathbf{x}^{(2)}(t) = \begin{pmatrix} -2e^t \\ e^t \end{pmatrix}.$$

(a) Calculate the Wronskian of $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$.

Solution.

$$W[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}] = \begin{vmatrix} 6 & -2e^t \\ t & e^t \end{vmatrix} = (6 + 2t)e^t.$$

(b) Where are $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ linearly independent?

Solution. The solutions are linearly independent wherever the Wronskian is not zero, *i.e.*, where $t \neq -3$.

(c) What conclusion can be drawn about the coefficients in the system of homogeneous differential equations satisfied by $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$?

Solution. Since the solutions are not linearly independent at $t = -3$, we expect that at least one of the coefficients of the system will be discontinuous at $t = -3$.

(d) By direct substitution, show that $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are solutions of

$$\dot{\mathbf{x}} = \frac{1}{t+3}\begin{pmatrix} t & -6 \\ (1-t)/2 & 4 \end{pmatrix}\mathbf{x}, \tag{6.1}$$

and hence verify your answer to (c).

Solution. Substituting the solutions into (6.1), we have

$$\begin{aligned}
\dot{\mathbf{x}}^{(1)} &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{t+3}\begin{pmatrix} t & -6 \\ (1-t)/2 & 4 \end{pmatrix}\begin{pmatrix} 6 \\ t \end{pmatrix} = \frac{1}{t+3}\begin{pmatrix} 6t-6t \\ 3(1-t)+4t \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
\dot{\mathbf{x}}^{(2)} &= \begin{pmatrix} -2e^t \\ e^t \end{pmatrix} = \frac{1}{t+3}\begin{pmatrix} t & -6 \\ (1-t)/2 & 4 \end{pmatrix}\begin{pmatrix} -2e^t \\ e^t \end{pmatrix} = \frac{1}{t+3}\begin{pmatrix} -2te^t - 6e^t \\ -(1-t)e^t + 4e^t \end{pmatrix} \\
&= \begin{pmatrix} -2e^t \\ e^t \end{pmatrix},
\end{aligned}$$

as required. Note that all the coefficients of (6.1) are discontinuous at $t = -3$, as surmised.

8. (BH) Consider the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -1 & -1 \\ 2 & -4 \end{pmatrix}\mathbf{x}.$$

(a) Show that the eigenvalues for this matrix are $\lambda_1 = -2$, $\lambda_2 = -3$.

Solution. Calculating the characteristic polynomial, we have

$$\begin{vmatrix} -1-\lambda & -1 \\ 2 & -4-\lambda \end{vmatrix} = (1+\lambda)(4+\lambda) + 2 = \lambda^2 + 5\lambda + 6 = (\lambda+2)(\lambda+3) = 0.$$

Thus we have the desired result.

(b) Find the general solution $\mathbf{x}(t)$ of this system.

Solution. Now we must calculate the eigenvectors corresponding to the eigenvalues. Solving for the first eigenvector, we obtain

$$(A + 2I)\mathbf{z}_1 = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}\mathbf{z}_1 = \mathbf{0}.$$

We note that the second row is twice the first, so the equations are redundant. Thus we must solve $x - y = 0$, so a typical eigenvector is $\mathbf{z}_1 = (1, 1)$. Solving for the second eigenvector, we obtain

$$(A + 3I)\mathbf{z}_2 = \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix}\mathbf{z}_2 = \mathbf{0}.$$

We note that the second row is minus the first, so the equations are redundant. Thus we must solve $2x - y = 0$, so a typical eigenvector is $\mathbf{z}_2 = (1, 2)$. Therefore, our solution is given by

$$\mathbf{x}(t) = c_1 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

(c) What happens to the solution as $t \to \infty$?

Solution. As $t \to \infty$, the exponentials decay to zero and we are left with

$$\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0}.$$

9. Consider the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -7 & 8 \\ -4 & 5 \end{pmatrix} \mathbf{x}. \tag{6.2}$$

(a) (BH) Find the solution to (6.2) subject to

$$\mathbf{x}(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$$

Solution. Calculating the characteristic polynomial, we have

$$\begin{vmatrix} -7-\lambda & 8 \\ -4 & 5-\lambda \end{vmatrix} = (-7-\lambda)(5-\lambda) + 32 = \lambda^2 + 2\lambda - 3 = (\lambda+3)(\lambda-1) = 0$$
$$\lambda_1 = -3, \quad \lambda_2 = 1.$$

Now we must calculate the eigenvectors corresponding to the eigenvalues. Solving for the first eigenvector, we obtain

$$(A + 3I)\mathbf{z}_1 = \begin{pmatrix} -4 & 8 \\ -4 & 8 \end{pmatrix} \mathbf{z}_1 = \mathbf{0}.$$

We note that the rows are the same, so the equations are redundant. Thus we must solve $-4x + 8y = 0$, so a typical eigenvector is $\mathbf{z}_1 = (2, 1)$. Solving for the second eigenvector, we obtain

$$(A - I)\mathbf{z}_2 = \begin{pmatrix} -8 & 8 \\ -4 & -4 \end{pmatrix} \mathbf{z}_2 = \mathbf{0}.$$

We note that the first row is twice the second, so the equations are redundant. Thus we must solve $-8x + 8y = 0$, so a typical eigenvector is $\mathbf{z}_2 = (1, 1)$. Therefore, the general solution is given by

$$\mathbf{x}(t) = c_1 e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{C}$$

Substituting $t = 0$ in (C) to find the initial conditions, we have

$$\begin{aligned} 2c_1 + c_2 &= 3 \\ c_1 + c_2 &= 0 \end{aligned} \qquad \Longrightarrow \qquad c_1 = 3, \quad c_2 = -3.$$

Therefore, the final solution is

$$\mathbf{x}(t) = 3e^{-3t}\begin{pmatrix}2\\1\end{pmatrix} - 3e^{t}\begin{pmatrix}1\\1\end{pmatrix}.$$

(b) (MP) Plot x_1 and x_2 from (a) for $t \in [0, 4]$.

(c) (BH) For a certain set of vectors $\mathbf{x}_0$, the solution to (6.2) with $\mathbf{x}(0) = \mathbf{x}_0$ decays to zero as $t \to \infty$. Determine $\mathbf{x}_0$.

Solution. In order for the solution to decay, $c_2 = 0$ in (A). Then plugging in $t = 0$, we have

$$\mathbf{x}(0) = \mathbf{x}_0 = c_1\begin{pmatrix}2\\1\end{pmatrix}.$$

(d) (MP) Choose an $\mathbf{x}_0$ which satisfies your answer to (c), then plot x_1 and x_2 for $t \in [0, 4]$.

10. (MP) Consider the system

$$\dot{\mathbf{x}} = \frac{1}{11}\begin{pmatrix}-47 & 2\\12 & -52\end{pmatrix}\mathbf{x}. \tag{6.3}$$

(a) Show that the eigenvalues for this system are $\lambda_1 = -4$, $\lambda_2 = -5$, and find the corresponding eigenvectors.

(b) Find the general solution $\mathbf{x}(t)$ of this system.

(c) Find the solution of the initial-value problem given by (6.3) and $\mathbf{x}(0) = (4, 0)$.

(d) Sketch the phase plane for this system.

In[ ]:=
```
Quit[]
```

HW1 (Checked)

HW2 (Checked)

HW3 (Checked)

HW4 (Checked)

HW5 (Checked)

HW6 (Checked)

Number 2b.

In[ ]:=
```
b4mat = {{-3, 6}, {1, -2}}
w1 = {-3, 1} * Exp[-5 * t]
w2 = t * w1
D[w2, t]
b4mat.w2 + w1
```

Out[ ]=

$\{\{-3, 6\}, \{1, -2\}\}$

Out[ ]=

$\{-3\, e^{-5\, t}, e^{-5\, t}\}$

Out[ ]=

$\{-3\, e^{-5\, t}\, t, e^{-5\, t}\, t\}$

Out[ ]=

$\{-3\, e^{-5\, t} + 15\, e^{-5\, t}\, t, e^{-5\, t} - 5\, e^{-5\, t}\, t\}$

Out[ ]=

$\{-3\, e^{-5\, t} + 15\, e^{-5\, t}\, t, e^{-5\, t} - 5\, e^{-5\, t}\, t\}$

Number 3d.

In[∘]:=
```
cmat = {{5, 4}, {1, 2}}
b5 = {7, -1}
```
Out[∘]=
```
{{5, 4}, {1, 2}}
```
Out[∘]=
```
{7, -1}
```

Check 5a.

In[∘]:=
```
Det[cmat]
```
Out[∘]=
```
6
```

Check 5b.

In[∘]:=
```
Inverse[cmat]
```
Out[∘]=

$$\left\{\left\{\frac{1}{3}, -\frac{2}{3}\right\}, \left\{-\frac{1}{6}, \frac{5}{6}\right\}\right\}$$

Check 5c.

In[∘]:=
```
Solve[cmat.{x, y} == b5, {x, y}]
```
Out[∘]=
```
{{x → 3, y → -2}}
```

Number 5d.

In[∘]:=
```
b5 = {{1, 3}, {4, 2}}
Eigensystem[b5]
```
Out[∘]=
```
{{1, 3}, {4, 2}}
```
Out[∘]=
```
{{5, -2}, {{3, 4}, {-1, 1}}}
```

Number 9b.

We define the solutions generally so we can easily use both parts.

In[∘]:=
```
sol2 = c1 * Exp[-3 * t] * {2, 1} + c2 * Exp[t] * {1, 1}
```
Out[∘]=

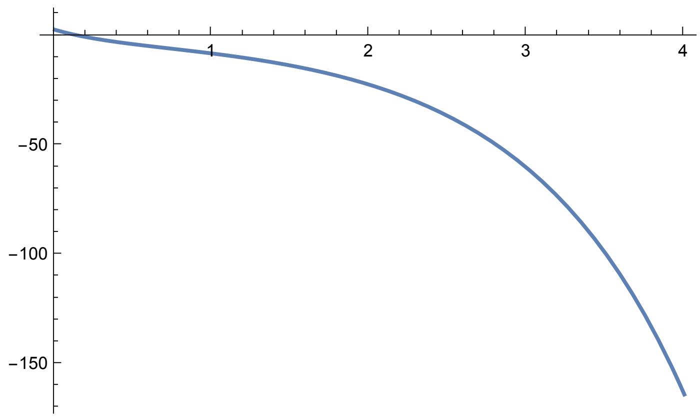
$$\left\{2\, c1\, e^{-3t} + c2\, e^{t},\ c1\, e^{-3t} + c2\, e^{t}\right\}$$

In[ ]:=
```
sol2b = sol2 /. {c1 → 3, c2 → -3}
Plot[sol2b[[1]], {t, 0, 4}]
Plot[sol2b[[2]], {t, 0, 4}]
```

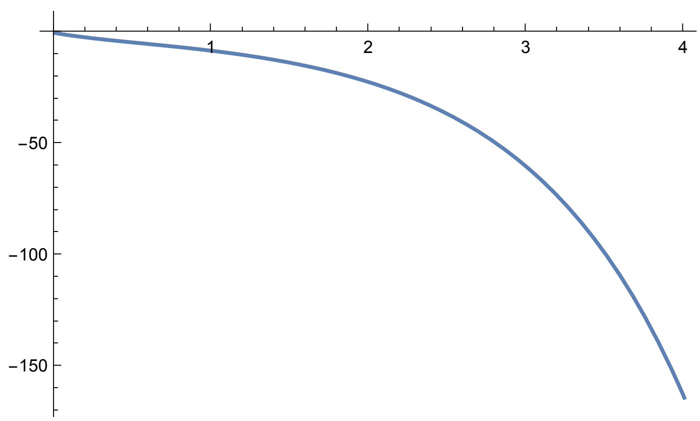
Out[ ]=

$\{6\,e^{-3t} - 3\,e^{t},\ 3\,e^{-3t} - 3\,e^{t}\}$

Out[ ]=



Out[ ]=



Number 9d.

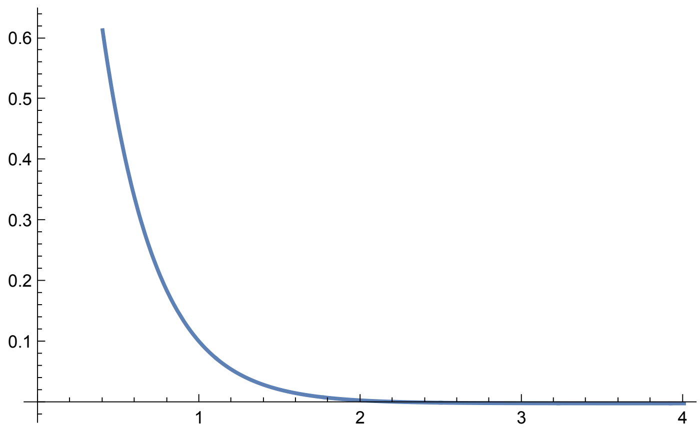
For simplicity, we choose c1=1.

In[ ]:=
```
sol2d = sol2 /. {c1 → 1, c2 → 0}
Plot[sol2d⟦1⟧, {t, 0, 4}]
Plot[sol2d⟦2⟧, {t, 0, 4}]
```

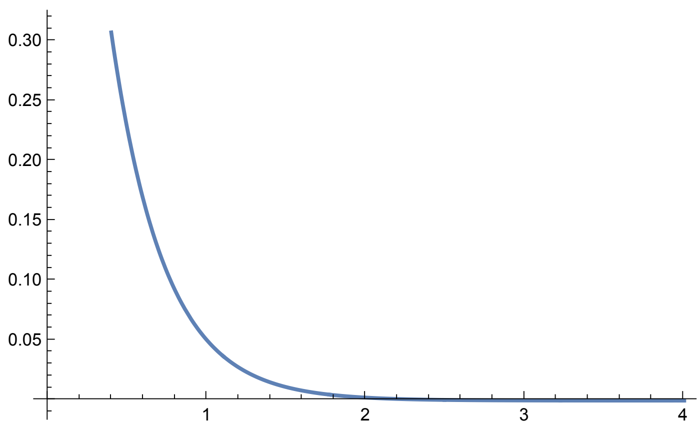
Out[ ]=

$$\left\{2\, e^{-3\,t},\ e^{-3\,t}\right\}$$

Out[ ]=



Out[ ]=



Number 10a.

In[ ]:=
```
b4mat = 1 / 11 * {{-47, 2}, {12, -52}}
Eigensystem[b4mat]
```

Out[ ]=

$$\left\{\left\{-\frac{47}{11},\ \frac{2}{11}\right\},\ \left\{\frac{12}{11},\ -\frac{52}{11}\right\}\right\}$$

Out[ ]=

$$\left\{\{-5,\ -4\},\ \left\{\left\{-\frac{1}{4},\ 1\right\},\ \left\{\frac{2}{3},\ 1\right\}\right\}\right\}$$

Number 10b.

In[ ]:=
```
xvec = {x[t], y[t]}
vecsys = D[xvec, t] == b4mat.xvec
DSolve[vecsys, xvec, t]
```

Out[ ]=

$\{x[t], y[t]\}$

Out[ ]=

$$\{x'[t], y'[t]\} == \left\{-\frac{47\,x[t]}{11} + \frac{2\,y[t]}{11}, \frac{12\,x[t]}{11} - \frac{52\,y[t]}{11}\right\}$$

Out[ ]=

$$\left\{\left\{x[t] \to \frac{1}{11} e^{-5t} \left(3 + 8 e^{t}\right) c_1 + \frac{2}{11} e^{-5t} \left(-1 + e^{t}\right) c_2, y[t] \to \frac{12}{11} e^{-5t} \left(-1 + e^{t}\right) c_1 + \frac{1}{11} e^{-5t} \left(8 + 3 e^{t}\right) c_2\right\}\right\}$$

Number 10c.

In[ ]:=
```
DSolve[{vecsys, (xvec /. (t → 0)) == {4, 0}}, xvec, t]
```

Out[ ]=

$$\left\{\left\{x[t] \to \frac{4}{11} e^{-5t} \left(3 + 8 e^{t}\right), y[t] \to \frac{48}{11} e^{-5t} \left(-1 + e^{t}\right)\right\}\right\}$$

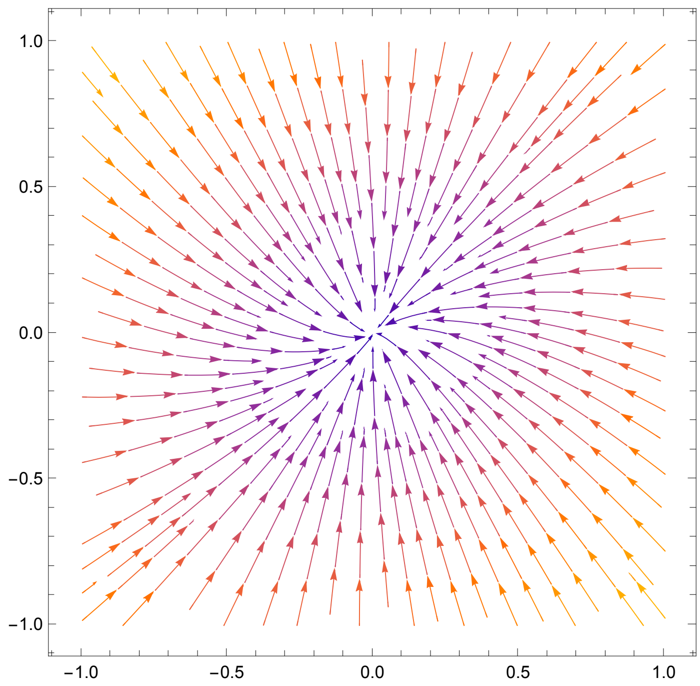
Number 10d.

In[∘]:=
```
tostream = b4mat.{x, y}
StreamPlot[tostream, {x, -1, 1}, {y, -1, 1}]
```

Out[∘]=

$$\left\{-\frac{47\,x}{11}+\frac{2\,y}{11},\ \frac{12\,x}{11}-\frac{52\,y}{11}\right\}$$

Out[∘]=



HW7 (Checked)

HW8 (Checked)

HW9 (Checked)

SSM (Checked)