Homework Set 4 Solutions

1. Consider the equation

$$\ddot{x} + 2\dot{x} + 5x = 0, \qquad x(0) = 1, \quad \dot{x}(0) = -(1 + 2\alpha), \quad \alpha \geq 0.$$

(a) (BH) Construct the solution $x(t)$ in standard form.

Solution. Substituting $x = e^{\lambda t}$, we obtain

$$\lambda^2 + 2\lambda + 5 = 0 \qquad \Longrightarrow \qquad \lambda_\pm = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm 2i,$$

so solutions are of the form $x(t) = e^{-t}(c_\mathrm{c} \cos 2t + c_\mathrm{s} \sin 2t)$. Solving the first initial condition, we immediately have that $x(0) = 1 = c_\mathrm{c}$. Solving the second initial condition, we have

$$\begin{aligned} \dot{x}(0) = e^{-t}\left(-2\sin 4t + 2c_\mathrm{s}\cos 4t\right) - e^{-t}\left(\cos 2t + c_\mathrm{s}\sin 2t\right)\big|_{t=0} &= -(1 + 2\alpha) \\ 2c_\mathrm{s} - 1 &= -1 - 2\alpha \\ c_\mathrm{s} &= -\alpha \end{aligned}$$

$$x(t) = e^{-t}(\cos 2t - \alpha \sin 2t). \tag{A.1}$$

(b) (BH) Convert your answer to (a) into magnitude-phase form.

Solution. Using the notation from class, we have from (A.1) that

$$\begin{aligned} c_\mathrm{c} = 1, \quad c_\mathrm{s} = -\alpha \qquad &\Longrightarrow \qquad A = \sqrt{1 + \alpha^2}, \\ x(t) = \sqrt{1 + \alpha^2}e^{-t}\cos(2t - \phi), \quad &\phi = \tan^{-1}(-\alpha) = -\tan^{-1}\alpha, \end{aligned} \tag{A.2}$$

where we have used the fact that $c_\mathrm{c} > 0$.

(c) (BH) Use your answers to (a) and (b) to confirm (twice) that $x(t_*) = 0$ whenever

$$\tan 2t_* = \frac{1}{\alpha}. \tag{4.1}$$

Solution. Using (A.1), we have

$$\begin{aligned} x(t_*) = e^{-t_*}[\cos 2t_* - \alpha \sin 2t_*] &= 0 \\ \cos 2t_* &= \alpha \sin 2t_* \\ \tan 2t_* &= \frac{1}{\alpha}. \end{aligned}$$

Using (A.2), we have

$$\begin{aligned} x(t_*) = \sqrt{1+\alpha^2}e^{-t_*}\cos(2t_* - \phi) &= 0 \\ 2t_* - \phi &= \frac{\pi}{2} \\ \tan 2t_* &= \tan\left(\phi + \frac{\pi}{2}\right) = -\cot\phi = -\frac{1}{\tan\phi} = \frac{1}{\alpha}, \end{aligned}$$

where we have used the definition of ϕ in (A.2).

(d) (MP) Plot (4.1).

(e) (BH) By considering your graph in the limits that $\alpha \to 0$ and $\alpha \to \infty$, construct upper and lower bounds on the smallest positive t_*.

Solution. We examine (4.1). As $\alpha \to \infty$, $\tan 2t_* \to 0$, so $t_* \to 0$. As $\alpha \to 0$, $\tan 2t_* \to \infty$, so $2t_* \to \pi/2$, and $t_* \to \pi/4$. Therefore, we have

$$0 < t_* \leq \frac{\pi}{4},$$

where we include equality only for the attainable limit $\alpha = 0$.

2. (BH) Find the general solution of

$$y^{(3)} - 3\ddot{y} + \dot{y} - 3y = 0.$$

Solution. Substituting $y = e^{\lambda t}$, we have

$$\begin{aligned} \lambda^3 - 3\lambda^2 + \lambda - 3 = (\lambda - 3)(\lambda^2 + 1) &= (\lambda - 3)(\lambda + i)(\lambda - i) = 0 \\ y(t) &= c_1 e^{3t} + c_{\mathrm{s}} \sin t + c_{\mathrm{c}} \cos t. \end{aligned}$$

3. (BH) Consider the differential equation

$$\mathcal{L}[y] = a\ddot{y} + b\dot{y} + cy = 0,$$

where the quadratic equation $a\lambda^2 + b\lambda + c = 0$ has the repeated root λ_1.

(a) Show that

$$\mathcal{L}[e^{\lambda t}] = a(\lambda - \lambda_1)^2 e^{\lambda t}. \tag{4.2}$$

Since the right side of (4.2) is zero when $\lambda = \lambda_1$, it follows that $e^{\lambda_1 t}$ is a solution of $\mathcal{L}[y] = 0$.

Solution. By the definition of $\mathcal{L}$, we have that

$$\mathcal{L}[e^{\lambda t}] = a\frac{d^2(e^{\lambda t})}{dt^2} + b\frac{d(e^{\lambda t})}{dt} + ce^{\lambda t} = (a\lambda^2 + b\lambda + c)e^{\lambda t}.$$

But $a\lambda^2 + b\lambda + c = 0$ has the repeated root λ_1, so it can be written as $a(\lambda - \lambda_1)^2$, and the proof is complete.

(b) Show that

$$\frac{\partial}{\partial\lambda}\mathcal{L}[e^{\lambda t}] = 2a(\lambda-\lambda_1)e^{\lambda t} + a(\lambda-\lambda_1)^2 te^{\lambda t}.$$

Since the right-hand side of the equation is zero when $\lambda = \lambda_1$, conclude that $te^{\lambda_1 t}$ is also a solution of $\mathcal{L}[y] = 0$.

Solution. Differentiating (4.2) with respect to r and interchanging differentiation as indicated, we have

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\mathcal{L}[e^{\lambda t}] &= \frac{\partial}{\partial\lambda}\left[a(\lambda-\lambda_1)^2 e^{\lambda t}\right] \\
\mathcal{L}\left[\frac{\partial(e^{\lambda t})}{\partial\lambda}\right] &= 2a(\lambda-\lambda_1)e^{\lambda t} + a(\lambda-\lambda_1)^2 te^{\lambda t} \\
\mathcal{L}[te^{\lambda t}] &= 2a(\lambda-\lambda_1)e^{\lambda t} + a(\lambda-\lambda_1)^2 te^{\lambda t}.
\end{aligned}$$

Since the right-hand side of the equation is zero when $\lambda = \lambda_1$, we have that conclude that $\mathcal{L}[te^{\lambda_1 t}] = 0$.

4. Consider the equation

$$y^{(4)} - 8\ddot{y} + 16y = 0. \tag{4.3}$$

(a) (BH) Find the general solution of (4.3).

Solution. Substituting $y = e^{\lambda t}$, we obtain

$$\begin{aligned}
\lambda^4 - 8\lambda^2 + 16 = (\lambda^2-4)^2 &= (\lambda+2)^2(\lambda-2)^2 = 0 \\
y(t) &= (c_1 + c_2 t)e^{-2t} + (c_3 + c_4 t)e^{2t}.
\end{aligned}$$

(b) (MP) Find the solution of (4.3) subject to

$$y(0) = 1, \quad \dot{y}(0) = -3, \quad \ddot{y}(0) = 5, \quad y^{(3)}(0) = -7.$$

5. (BH) Find the general solution to the differential equation

$$3\ddot{y} + 5\dot{y} - 2y = -2t^2 + 10t. \tag{4.4}$$

Solution. Substituting $x = e^{\lambda t}$ into the homogeneous problem, we obtain

$$3\lambda^2 + 5\lambda - 2 = (3\lambda-1)(\lambda+2) = 0 \qquad \Longrightarrow \qquad \lambda = 1/3, -2,$$

so the homogeneous solution is given by

$$y_{\mathrm{h}} = c_1 e^{t/3} + c_2 e^{-2t}.$$

The form of the right-hand side motivates a substitution of the form

$$y_{\mathrm{p}} = a_2 t^2 + a_1 t + a_0.$$

Substituting this into (4.4), we obtain

$$\begin{aligned} 3(2a_2) + 5(2a_2 t + a_1) - 2(a_2 t^2 + a_1 t + a_0) &= -2t^2 + 10t \\ t^2(-2a_2 + 2) + t(10a_2 - 2a_1 - 10) + 6a_2 + 5a_1 - 2a_0 &= 0. \end{aligned} \tag{B}$$

We solve for the a_j by zeroing out the coefficients of the t terms. Starting with zeroing out the t^2 terms, we have

$$-2a_2 + 2 = 0 \qquad \Longrightarrow \qquad a_2 = 1.$$

Substituting this result into (B), we obtain, zeroing out the t and constant terms,

$$\begin{aligned} -2a_1 t + 6 + 5a_1 - 2a_0 &= 0 \qquad \Longrightarrow \qquad a_1 = 0 \\ 6 - 2a_0 &= 0 \qquad \Longrightarrow \qquad a_0 = 3 \\ y_{\text{p}} &= t^2 + 3 \\ y &= c_1 e^{t/3} + c_2 e^{-2t} + t^2 + 3. \end{aligned}$$

6. Consider the differential equation

$$\ddot{y} - \omega^2 y = e^t + e^{-t}. \tag{4.5}$$

(a) (BH) Find the general solution to (4.5) Be sure to account for all $\omega \neq 0$.

Solution. Using the method of undetermined coefficients, we try to find a particular solution of the form

$$y_{\text{p}} = c_+ e^t + c_- e^{-t}.$$

Substituting in this form, we obtain

$$\begin{aligned} c_+ e^t + c_- e^{-t} - \omega^2(c_+ e^t + c_- e^{-t}) &= e^t + e^{-t} \\ c_+(1 - \omega^2)e^t + c_-(1 - \omega^2)e^{-t} &= e^t + e^{-t} \end{aligned}$$

$$c_+ = c_- = \frac{1}{1 - \omega^2}, \qquad \omega \neq \pm 1.$$

For the case where $\omega = \pm 1$, we try

$$y_{\text{p}} = a_+ t e^t + a_- t e^{-t}.$$

Substituting in this form, we obtain

$$\begin{aligned} a_+(t + 2)e^t + a_-(t - 2)e^{-t} - (a_+ t e^t + a_- t e^{-t}) &= e^t + e^{-t} \\ 2(a_+ e^t - a_- e^{-t}) &= e^t + e^{-t} \end{aligned}$$

$$a_+ = \frac{1}{2}, \qquad a_- = -\frac{1}{2}.$$

To obtain the homogeneous solution, we try $y_\mathrm{h} = e^{\lambda t}$, which yields

$$\lambda^2 - \omega^2 = 0,$$
$$y_\mathrm{h} = Ae^{\omega t} + Be^{-\omega t},$$

as long as $\omega \neq 0$ so we don't have a double root. Therefore, the general solution is given by

$$y(t) = y_\mathrm{p}(t) + Ae^{\omega t} + Be^{-\omega t}, \qquad y_\mathrm{p}(t) = \begin{cases} \dfrac{e^t + e^{-t}}{1-\omega^2} = \dfrac{2\cosh t}{1-\omega^2}, & \omega \neq \pm 1, \\ \dfrac{t(e^t - e^{-t})}{2} = t\sinh t, & \omega = \pm 1. \end{cases}$$

(b) (MP) Solve (4.5). Does Mathematica miss anything?

7. Consider the equations

$$6\ddot{y} + 5\dot{y} + y = 20\cos^2\left(\frac{t}{2}\right), \qquad y(0) = 14, \qquad \dot{y}(0) = -1, \tag{4.6a}$$
$$6\ddot{y} + 5\dot{y} + y = 20\cos^4\left(\frac{t}{2}\right), \qquad y(0) = 14, \qquad \dot{y}(0) = -1. \tag{4.6b}$$

(a) (BH) Find the solution to (4.6a).

Solution. $\cos^2(t/2) = (1+\cos t)/2$, so we have

$$6\ddot{y} + 5\dot{y} + y = 10(1+\cos t)$$

and thus we try a particular solution of the form

$$y_\mathrm{p} = c_c \cos t + c_s \sin t + c_0.$$

Substituting in this form, we obtain

$$-6c_c\cos t - 6c_s \sin t - 5c_c \sin t + 5c_s\cos t + (c_c\cos t + c_s \sin t + c_0) = 10(1+\cos t)$$
$$5(c_s - c_c)\cos t - 5(c_s + c_c)\sin t + c_0 = 10(1+\cos t)$$

We solve for the constants by matching up the constant terms, as well as the coefficients of $\sin t$ and $\cos t$:

$$c_0 = 10 \qquad \text{(constant)}$$
$$c_s - c_c = 2 \qquad (\cos t)$$
$$c_s + c_c = 0. \qquad (\sin t)$$

Solving the last two equations together, we have $c_s = 1$, $c_c = -1$. By substituting $y = e^{\lambda t}$, we can obtain the homogeneous solution, where λ solves

$$6\lambda^2 + 5\lambda + 1 = (3\lambda+1)(2\lambda+1) = 0 \qquad \Longrightarrow \qquad \lambda = -\frac{1}{3}, -\frac{1}{2}.$$

Thus, we have

$$y(t) = \sin t - \cos t + 10 + Ae^{-t/3} + Be^{-t/2}.$$

Solving the initial data, we obtain

$$\begin{aligned} 9 + A + B &= 14 = y(0) \\ 1 - \frac{A}{3} - \frac{B}{2} &= -1 = \dot{y}(0). \end{aligned} \qquad \Longrightarrow \qquad \begin{aligned} A + B &= 5 \\ 2A + 3B &= 12. \end{aligned}$$

Solving these equations together, we have that $A = 3$, $B = 2$, so the solution is

$$y(t) = \sin t - \cos t + 10 + 3e^{-t/3} + 2e^{-t/2}.$$

(b) (MP) Find the solution to (4.6b).
(c) (MP) Plot the solutions to (4.6a) and (4.6b) on the same graph for $t \in [0, 10\pi]$. Why should the graphs be so similar?

8. (BH) Find the general solution to the differential equation

$$\ddot{y} - \omega^2 y = e^t + e^{-t}.$$

Be sure to account for all $\omega \neq 0$.

Solution. This is the same problem as #6, so we know that the homogeneous solutions are given by

$$y_1 = e^{\omega t}, \qquad y_2 = e^{-\omega t}$$

as long as $\omega \neq 0$. Then the Wronskian is given by

$$W = \begin{vmatrix} e^{\omega t} & e^{-\omega t} \\ \omega e^{\omega t} & -\omega e^{-\omega t} \end{vmatrix} = -2\omega.$$

Using the variation of parameters formula, we have

$$\begin{aligned} y_{\text{p}}(t) &= -e^{\omega t} \int \frac{e^{-\omega t}(e^t + e^{-t})}{(-2\omega)}\, dt + e^{-\omega t} \int \frac{e^{\omega t}(e^t + e^{-t})}{(-2\omega)}\, dt \qquad \text{(C)} \\ &= \frac{e^{\omega t}}{2\omega}\left[\frac{e^{(1-\omega)t}}{1-\omega} - \frac{e^{-(1+\omega)t}}{1+\omega}\right] - \frac{e^{-\omega t}}{2\omega}\left[\frac{e^{(1+\omega)t}}{1+\omega} - \frac{e^{(\omega-1)t}}{\omega - 1}\right] \\ &= \frac{e^t}{2\omega}\left(\frac{1}{1-\omega} - \frac{1}{1+\omega}\right) + \frac{e^{-t}}{2\omega}\left(\frac{1}{1-\omega} - \frac{1}{1+\omega}\right) = \frac{e^t + e^{-t}}{1-\omega^2}, \qquad \omega \neq \pm 1. \end{aligned}$$

If $\omega = \pm 1$, we see that (C) becomes

$$\begin{aligned} y_{\text{p}}(t) &= -e^{\omega t} \int \frac{1 + e^{-2\omega t}}{(-2\omega)}\, dt + e^{-\omega t} \int \frac{1 + e^{2\omega t}}{(-2\omega)}\, dt = \frac{e^{\omega t}}{2\omega}\left(t - \frac{e^{-2\omega t}}{2\omega}\right) - \frac{e^{-\omega t}}{2\omega}\left(t + \frac{e^{2\omega t}}{2\omega}\right) \\ &= \frac{t(e^{\omega t} - e^{-\omega t})}{2\omega} = \frac{t(e^t - e^{-t})}{2}. \end{aligned}$$

Therefore, the general solution is given by

$$y(t) = y_{\mathrm{p}}(t) + Ae^{\omega t} + Be^{-\omega t}, \qquad y_{\mathrm{p}}(t) = \begin{cases} \dfrac{e^t + e^{-t}}{1-\omega^2} = \dfrac{2\cosh t}{1-\omega^2}, & \omega \neq \pm 1, \\ \dfrac{t(e^t - e^{-t})}{2} = t\sinh t, & \omega = \pm 1, \end{cases}$$

as in #6.

9. (BH) Find the general solution of

$$\ddot{y} - 6\dot{y} + 9y = \frac{e^{3t}}{t}.$$

Solution. Substituting $y = e^{\lambda t}$ into the homogeneous form of the equation, we have

$$\lambda^2 - 6\lambda + 9 = (\lambda - 3)^2 = 0.$$

Since we have a double root, the solutions are $y_1 = e^{3t}$ and $y_2 = te^{3t}$, which have the Wronskian

$$\begin{vmatrix} e^{3t} & te^{3t} \\ 3e^{3t} & (1+3t)e^{3t} \end{vmatrix} = e^{6t}.$$

Then using the formula from class, we have that a particular solution is given by

$$y_{\mathrm{p}}(t) = -e^{3t}\int \frac{te^{3t}}{e^{6t}}\frac{e^{3t}}{t}\,dt + te^{3t}\int \frac{e^{3t}}{e^{6t}}\frac{e^{3t}}{t}\,ds = -e^{3t}t + te^{3t}\log t.$$

Thus the general solution is given by the homogenous solution plus the particular solution:

$$y(t) = e^{3t}(c_1 + c_2 t + t\log t).$$

where we have folded the $-e^{3t}t$ term in the particular solution into the arbitrary constant c_2.

10. Consider the differential equation

$$\ddot{g} + 4g = \sec 2t, \qquad g(0) = 0, \qquad \dot{g}(0) = 0.$$

(a) (BH) Where is this equation guaranteed to have a unique solution?

Solution. $\sec 2t$ is undefined whenever $\cos 2t = 0$, or when $t = (2n+1)\pi/4$, n an integer. Since the initial conditions were given at $t = 0$, we see that the solution has a unique solution when $t \in (-\pi/4, \pi/4)$.

(b) (BH) Show that the solution is given by

$$g(t) = \frac{t\sin 2t}{2} + \frac{\log(\cos 2t)\cos(2t)}{4}. \tag{4.7}$$

Be sure to check the initial conditions.

Solution. Substituting $y = e^{\lambda t}$ into the homogeneous form of the equation, we have

$$\lambda^2 + 4 = 0 \qquad \Longrightarrow \qquad \lambda = \pm 2i,$$

so

$$g_1 = \sin 2t, \qquad g_2 = \cos 2t, \qquad W = \begin{vmatrix} \sin 2t & \cos 2t \\ 2\cos 2t & -2\sin 2t \end{vmatrix} = -2.$$

Then using the variation of parameters formula, we have

$$\begin{aligned} g_{\mathrm{p}}(t) &= -\sin 2t \int \frac{\cos 2t \sec 2t}{(-2)}\, dt + \cos 2t \int \frac{\sin 2t \sec 2t}{(-2)}\, dt \\ &= \frac{\sin 2t}{2} \int dt + \frac{\cos 2t}{2} \int \frac{-\sin 2t}{\cos 2t}\, dt = \frac{\cos 2t}{2} \frac{\log(\cos 2t)}{2} + \frac{t \sin 2t}{2} \end{aligned}$$

as required. Then using the variation of parameters formula, we have

$$\begin{aligned} g_{\mathrm{p}}(t) &= \int_0^t \frac{\sin 2s \cos 2t - \sin 2t \cos 2s}{(-2)} \sec 2s\, ds = \frac{\cos 2t}{2} \int_0^t \frac{-\sin 2s}{\cos 2s}\, ds + \frac{\sin 2t}{2} \int_0^t ds \\ &= \frac{\cos 2t}{2} \frac{[\log(\cos 2s)]_0^t}{2} + \frac{[s]_0^t \sin 2t}{2} = \frac{t \sin 2t}{2} + \frac{\cos 2t}{2} \frac{\log(\cos 2t)}{2}. \end{aligned}$$

This is exactly the solution in (4.7), but to verify we must check the initial conditions:

$$\begin{aligned} g(0) &= 0 + \frac{\log 1}{4} = 0, \\ \dot{g}(0) &= \frac{\sin 2t + 2t \cos 2t}{2} + \frac{1}{4} \left[-2 \sin 2t \log(\cos t) + \cos 2t \frac{-2 \sin 2t}{\cos 2t} \right] \Bigg|_{t=0} = 0. \end{aligned}$$

(c) (MP) Show that this solution has no extrema for $t > 0$.

In[ ]:=
```
Quit[]
```

HW1 (Checked)

HW2 (Checked)

HW3 (Checked)

HW4 (Checked)

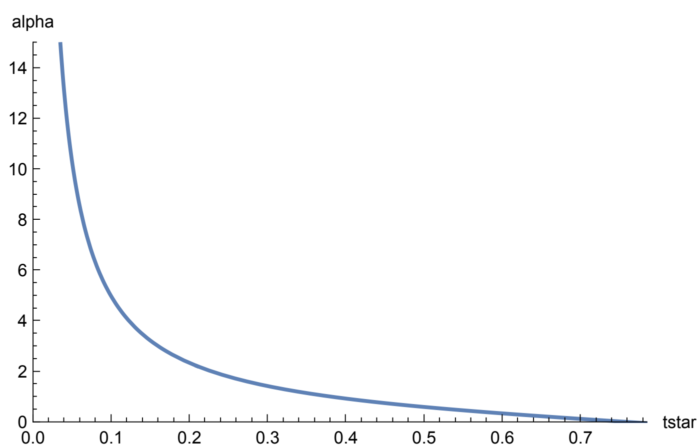
Number 1c.

In[1]:=
```
eq3 = Tan[2 * tstar] == 1 / alpha
Solve[eq3, alpha]
Plot[alpha /. %, {tstar, 0, Pi / 4},
 PlotRange → {{0, Pi / 4}, {0, 15}}, AxesLabel → {tstar, alpha}]
```

Out[1]= $\text{Tan}[2\ \text{tstar}] == \frac{1}{\text{alpha}}$

Out[2]= {{alpha → Cot[2 tstar]}}

Out[3]=



Number 4b.

In[ ]:=
```
sys10 = {D[y[t], {t, 4}] - 8 * y''[t] + 16 * y[t] == 0,
  y[0] == 1, y'[0] == -3, y''[0] == 5, (D[y[t], {t, 3}] /. t → 0) == -7}
DSolve[sys10, y[t], t]
```

Out[ ]=

$$\begin{pmatrix} 16\,y[t] - 8\,y''[t] + y^{(4)}[t] == 0 \\ y[0] == 1 \\ y'[0] == -3 \\ y''[0] == 5 \\ y^{(3)}[0] == -7 \end{pmatrix}$$

Out[ ]=

$$\left(y[t] \to \frac{1}{32}\, e^{-2t} \left(45 - 13\, e^{4t} + 6\,t + 14\, e^{4t}\, t\right) \right)$$

Number 6b.

In[ ]:=
```
eq6 = y''[t] - omega^2 * y[t] == Exp[t] + Exp[-t]
DSolve[eq6, y[t], t]
Simplify[%]
```

Out[ ]=

$$-\text{omega}^2\, y[t] + y''[t] == e^{-t} + e^{t}$$

Out[ ]=

$$\left(y[t] \to -\frac{e^{-\text{omega}\,t-(1+\text{omega})\,t} \left(-e^{2\,\text{omega}\,t} - e^{2\,(1+\text{omega})\,t} + e^{2\,t+2\,\text{omega}\,t} + e^{(-1+\text{omega})\,t+(1+\text{omega})\,t} + e^{2\,\text{omega}\,t}\,\text{omega} + e^{2\,(1+\text{omega})\,t}\,\text{omega} + e^{2\,t+2\,\text{omega}\,t}\,\text{o}\right.}{2\,(-1+\text{omega})\,\text{omega}\,(1+\text{omega})}$$

Out[ ]=

$$\left(y[t] \to \frac{e^{-((1+2\,\text{omega})\,t)} \left(-e^{2\,\text{omega}\,t} - e^{2\,(1+\text{omega})\,t} + e^{t+3\,\text{omega}\,t}\left(-1+\text{omega}^2\right) c_1 + e^{(1+\text{omega})\,t}\left(-1+\text{omega}^2\right) c_2\right)}{-1+\text{omega}^2} \right)$$

Mathematica doesn’t recognize that there is a special case when omega^2=1.

Number 7b.

In[ ]:=
```
eq5b = {6 * y''[t] + 5 * y'[t] + y[t] == 20 * Cos[t / 2] ^ 4, y[0] == 14, y'[0] == -1}
sol5b = DSolve[eq5b, y[t], t]
```

Out[ ]=

$$\begin{pmatrix} y[t] + 5\,y'[t] + 6\,y''[t] == 20\,\text{Cos}\left[\frac{t}{2}\right]^4 \\ y[0] == 14 \\ y'[0] == -1 \end{pmatrix}$$

Out[ ]=

$$\left(y[t] \to -\frac{e^{-t/2}\left(3404 - 12\,954\,e^{t/6} - 9435\,e^{t/2} + 1258\,e^{t/2}\,\text{Cos}[t] + 115\,e^{t/2}\,\text{Cos}[2\,t] - 1258\,e^{t/2}\,\text{Sin}[t] - 50\,e^{t/2}\,\text{Sin}[2\,t]\right)}{1258} \right)$$

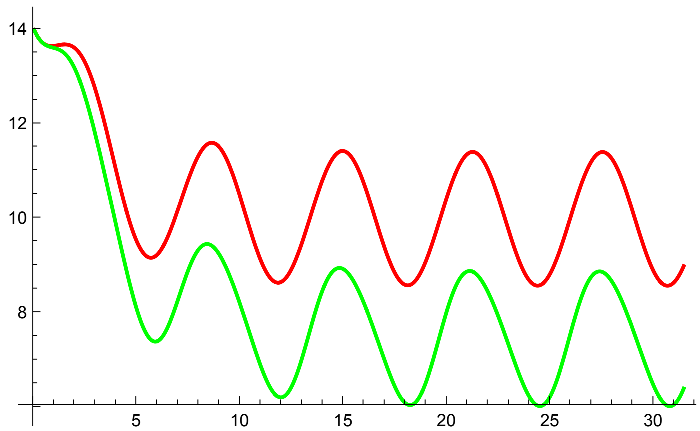
Number 7c.

In[ ]:=
```
sol5a = 2 * Exp[-1 / 2 * t] + 3 * Exp[-1 / 3 * t] - Cos[t] + Sin[t] + 10
Plot[{sol5a, y[t] /. sol5b}, {t, 0, 10 * Pi}, PlotStyle → {Red, Green}]
```

Out[ ]=

$10 + 2\,e^{-t/2} + 3\,e^{-t/3} - \text{Cos}[t] + \text{Sin}[t]$

Out[ ]=



The graphs look so similar since $\cos(t^2)$ and $\cos(t^4)$ are both positive, vary on the same time scale, and vary only slightly in their amplitudes.

Number 10c.

In[ ]:=
```
sol8 = 1 / 2 * t * Sin[2 * t] + 1 / 4 * Log[Cos[2 * t]] * Cos[2 * t]
```

Out[ ]=

$$\frac{1}{4}\text{Cos}[2\,t]\,\text{Log}[\text{Cos}[2\,t]] + \frac{1}{2}\,t\,\text{Sin}[2\,t]$$

To show that there is no root, we take the derivative of this expression and then use the `FindRoot` command to try to find a root. It returns an error because there isn't one, and hence there isn't an extremum in the region of interest.

In[ ]:=
```
D[sol8, t]
FindRoot[% == 0, {t, 0.01, 0.001, Pi / 4}]
```

Out[ ]=

$$t\,\text{Cos}[2\,t] - \frac{1}{2}\,\text{Log}[\text{Cos}[2\,t]]\,\text{Sin}[2\,t]$$

FindRoot: The point {0.001} is at the edge of the search region {0.001, 0.785398} in coordinate 1 and the computed search direction points outside the region.

Out[ ]=

{t → 0.001}

HW5 (Checked)