Updates

1. Exam III will be administered on Friday, Nov. 22. You will need to bring a small blue book, as well as your laptop.
2. The exam will cover up through section 5.4, so so the first five homework problems on this set for practice beforehand.

Homework Set 9

Read sections 5.4, 6.5, 6.6, 10.2, 10.3, 10.5.

Sections 6.5/6.6

1. (BH) Consider the following problem:

$$\ddot{y} - (a+b)\dot{y} + aby = f(t), \qquad y(0) = 0, \quad \dot{y}(0) = 0, \quad a \neq b.$$

   (a) Solve the problem using Laplace transforms.
   (b) What is the Green's function for this problem?

2. Consider the Laplace transform given by

$$\hat{f}(s) = \frac{1}{s^2(s^2+4)}.$$

   (a) (BH) Evaluate the inverse Laplace transform by using a convolution involving the transform of the sine. (*Hint: Use integration by parts.*)
   (b) (MI) Evaluate the inverse Laplace transform by using a convolution involving the transform of the cosine. You must set up the integral by hand, but you can use Mathematica to actually integrate. Verify that your answer matches (a).
   (c) (MP) Invert the Laplace transform directly using Mathematica.

3. (BH) Solve the following problem using Laplace transforms:

$$\ddot{y} + 16y = f(t), \qquad y(0) = a, \quad \dot{y}(0) = b.$$

Section 5.4

4. (BH) Find the solution of the following Euler equations:

   (a)
$$x^2y'' + 5xy' + 5y = 0.$$

   (b)
$$x^2y'' + 5xy' + 4y = 0, \qquad y(1) = 1, \quad y'(1) = 1.$$

5. (BH) Consider the following forced Euler equation:
$$ax^2y'' + bxy' + cy = x^\alpha.$$

   (a) Try a particular solution of the form $y_\text{p} = Ax^\alpha$, as in the method of undetermined coefficients. Explain why this approach works.

   (b) Use your approach in (a) to write the general solution of
$$x^2y'' - xy' - 3y = 6x^2.$$

Section 10.5

6. Consider the heat conduction problem

$$\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial u}{\partial t}, \qquad 0 < x < \pi, \quad t > 0; \tag{9.1a}$$
$$u(0,t) = 0, \qquad u(\pi,t) = 0, \quad t > 0; \tag{9.1b}$$
$$u(x,0) = \sin 3x + 2\sin 7x, \quad 0 \le x \le \pi. \tag{9.1c}$$

   (a) (BH) Find the solution to (9.1).

   (b) (MP) Graph your solution for $t = 0$, 1, and 2. What happens to the number of oscillations as t gets larger? (*Hint: Look at the time dependence of your solution.*)

7. (BH) The wave equation is given by
$$c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}.$$

   Assuming that $u(x,t) = T(t)X(x)$, find ordinary differential equations satisfied by T and X.

Section 10.2

8.
   (a) (BH) exercise 15(a)
   (b) (BH) exercise 15(b)
   (c) (MP) Using your answer to (b), plot $s_m(x)$ (book notation for the first m terms in the Fourier series) for $m = 15$ and $x \in [-3\pi, 3\pi]$.

9. (MP) Find the first five terms in the Fourier series of

$$f(x) = xe^{-x}, \qquad x \in [-\pi, \pi].$$

Section 10.3

10. (BH) exercise 14