

Phase Plane: Complex Eigenvalues

For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -2 & -6 \\ 3 & 4 \end{pmatrix} \mathbf{x}, \quad (2)$$

the solution is

$$\mathbf{x} = c_1 e^t \begin{pmatrix} -\cos 3t - \sin 3t \\ \cos 3t \end{pmatrix} + c_2 e^t \begin{pmatrix} \cos 3t - \sin 3t \\ \sin 3t \end{pmatrix}.$$

Since the real part of the eigenvalues is positive, we have an unstable spiral, as shown below.

