MATH 302-011 | Ordinary Differential Equations
Prof. D. A. Edwards | Sept. 25, 2024

Amplitude-Phase Form

In class we showed that

$$y_+(t) = e^{-t/2}(\cos t - \sin t) = (e^{-t/2}\sqrt{2})\cos\left(t + \frac{\pi}{4}\right),$$

where the term in the parentheses is the amplitude. A graph is shown below.

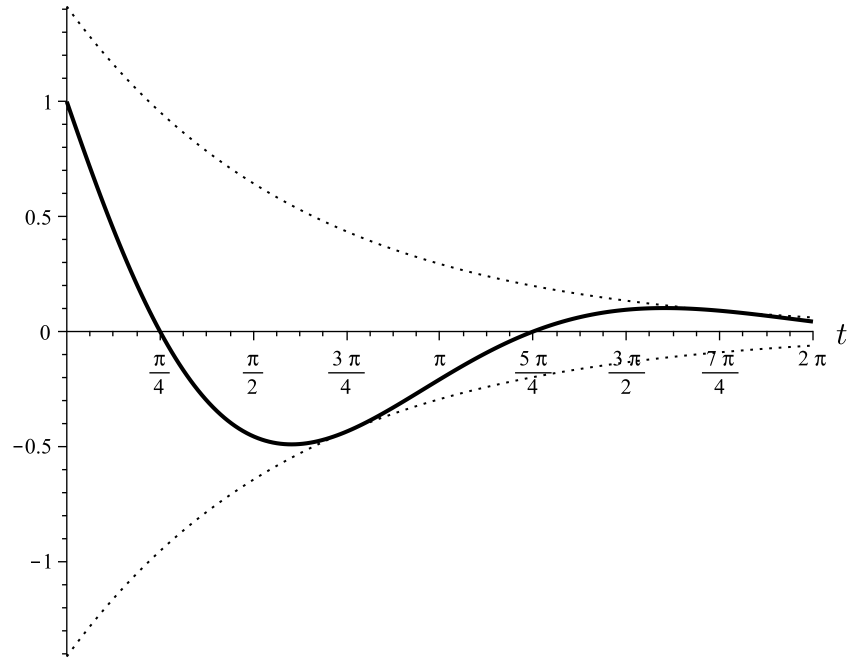

Figure 1. Solid curve: $y_+(t)$. Dotted curve: amplitude $(e^{-t/2}\sqrt{2})$.

Note that the graph touches the amplitude curve (the *envelope*) when $t = 3\pi/4$ and $t = 7\pi/4$, which correspond to the min and max of the cosine, given that it's been shifted.

In class we showed that

$$y_-(t) = -y_+(t) = -e^{-t/2}(\cos t - \sin t) = (e^{-t/2}\sqrt{2})\cos\left(t - \frac{3\pi}{4}\right),$$

where the term in the parentheses is the amplitude. Note that the amplitude is the same as y_-; the negative sign is handled by a change in the phase angle. A graph is shown below.

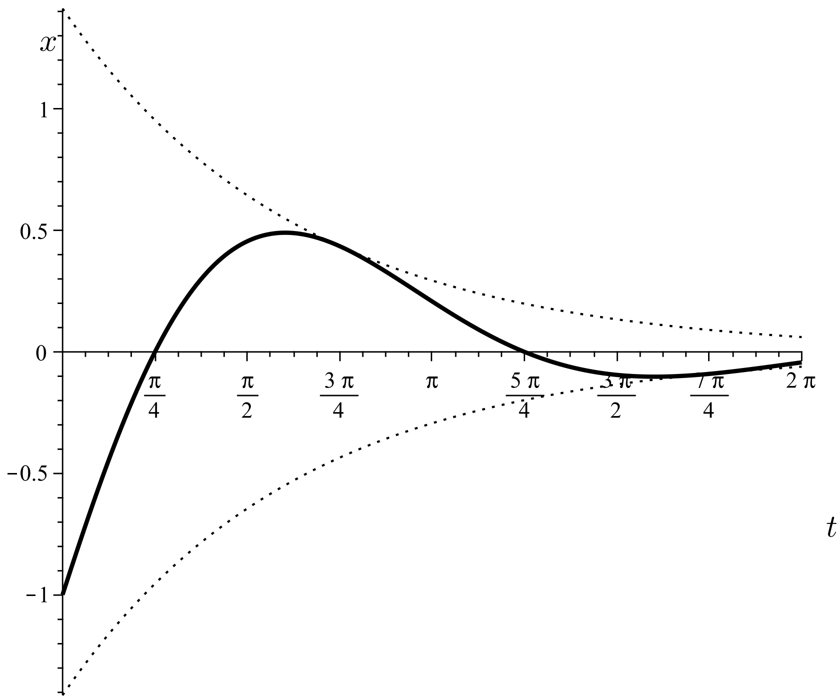

Figure 2. Solid curve: $y_-(t)$. Dotted curve: amplitude $(e^{-t/2}\sqrt{2})$.

Note that the graph touches the amplitude curve (the *envelope*) when $t = 3\pi/4$ and $t = 7\pi/4$, which correspond to the max and min of the cosine, given that it's been shifted.