# MEMS DIFFRACTION GRATING

M. Gutin - Interscience

J. Castricane – Interscience

J. D. Cole RPI
J. Haus RPI
O. Ryzhov RPI
G. Shavlov RPI

**Aim:** Simple formulas for light diffracted in various orders by dynamic gratings, as designed and made by Interscience.

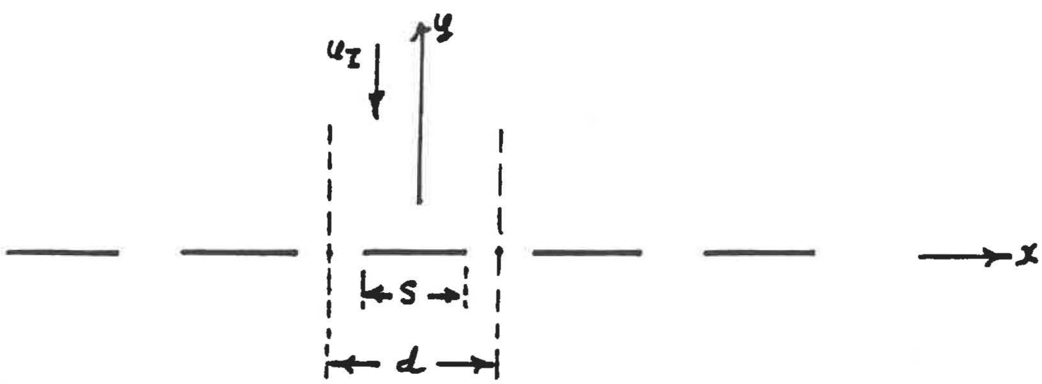
Problem 1. Diffraction by a regular grating - electric field parallel to rulings. (P-case)

incoming light $\quad E_z = u_I e^{-i\omega t},\ u_I = e^{-iky}$ $\quad$ $(Fig.1.)$

diffracted field $\quad E_z = u(x,y)e^{-i\omega t}$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0 \quad (A), \qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$c$=velocity of light, $\quad \lambda$ = wave length, $\quad k$ = wave number




The rulings have width $s$, spacing $d$; solution has period $d$ in $x$. In the general theory of Helmholtz eqn. (A) the solution is represented by distributions of dipoles and sources over the rulings. An integral equation results depend on the nature of the boundary condition. Here we bypass a rigorous treatment in order to make a comparative study assuming that the diffracted field in $y > 0$ is produced only by electric dipoles on the surface. We will also assume that the dipole strength is constant over the ruling. $|x| < s/2$.

That is, over the basic period $| x |< d/_2$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = D(x)\delta'(y)$$

where

$$D(x) = D_0 = \text{const} \qquad | x |< \frac{s}{2} \qquad \text{period } d.$$

$$= 0 \qquad | x |> \frac{s}{2}$$

For $D(x)$ we have a Fourier Series

$$D(x) = \sum_{n=-\infty}^{\infty} \widetilde{D}_n \, e^{i\alpha_n x} \qquad \alpha_n = \frac{n(2\pi)}{d}$$

with coefficients

$$\widetilde{D}_n = \frac{D_0}{d}\int_{-s/2}^{s/2} e^{-i\alpha_n x}\, dx = \frac{2D_0}{d}\int_0^{\frac{s}{2}} \cos\alpha_n x dx = \frac{2D_0}{d}\,\frac{\sin\alpha_n x}{\alpha_n} = \frac{D_0}{\pi}\,\frac{\sin\frac{n\pi s}{d}}{n}$$

Therefore we have a Fourier Series

$$u(x,y) = \sum_{n=-\infty}^{\infty} \widetilde{u}_n(y) e^{i\alpha_n x}$$

$$\frac{d^2\widetilde{u}_n}{dy^2} + (k^2 - \alpha_n^2)\widetilde{u}_n = \widetilde{D}_n \delta'(y) \qquad | x |< \frac{d}{2} \qquad y \geq 0$$

The solution for $y \geq 0$ which has outgoing waves ($+y$ direction) is

$$\widetilde{u}_n = \frac{1}{2}\,\widetilde{D}_n\, e^{i\sqrt{k^2 - \alpha_n^2}\, y}$$

Thus the diffracted field is

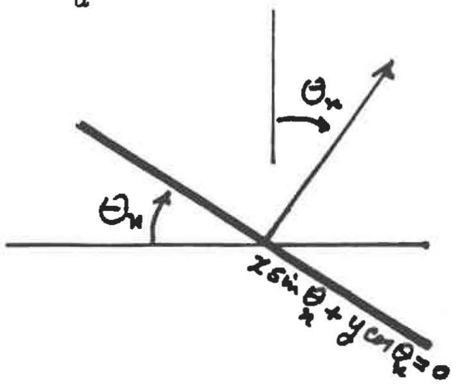
$$u(x,y) = \frac{D_0}{2\pi}\sum_{n=-\infty}^{\infty} \frac{\sin\frac{n\pi s}{d}}{n}\, e^{i\alpha_n x + i\sqrt{k^2 - \alpha_n^2}\, y}$$

For $\alpha_n > k$ the waves damp rather than propagate. $D_o$ may be complex, that is, cause a phase shift. The $n^{th}$ order of diffraction occurs at angle $\Theta_n$ to the normal where

$$k \sin\Theta_n = \alpha_n, \qquad k\cos\Theta_n = \sqrt{k^2 - \alpha_n^2} \qquad \text{(Fig.2.)}$$

$$\sin\Theta_n = \frac{\alpha_n}{k} = \frac{n(2\pi)}{d}\frac{\lambda}{2\pi} = n\frac{\lambda}{d} < 1, \; n = 1, 2, \cdots [\frac{d}{\lambda}]$$

$$[\frac{\lambda}{d}] = \text{largest integer less than } \frac{\text{wave}-\text{length}}{\text{period}}$$




The amplitude of the $n^{th}$ order of diffraction is proportinal to $|D_0 \frac{\sin\frac{n\pi s}{d}}{n}|$

Fig. 2.

For a typical Interscience example $\left(\lambda = 7\mu,\ d = 5\mu,\ \frac{\lambda}{d} \sim \frac{1}{7},\ [\frac{d}{\lambda}] = 7.\right)$ seven orders can be observed with qualitative amplitude

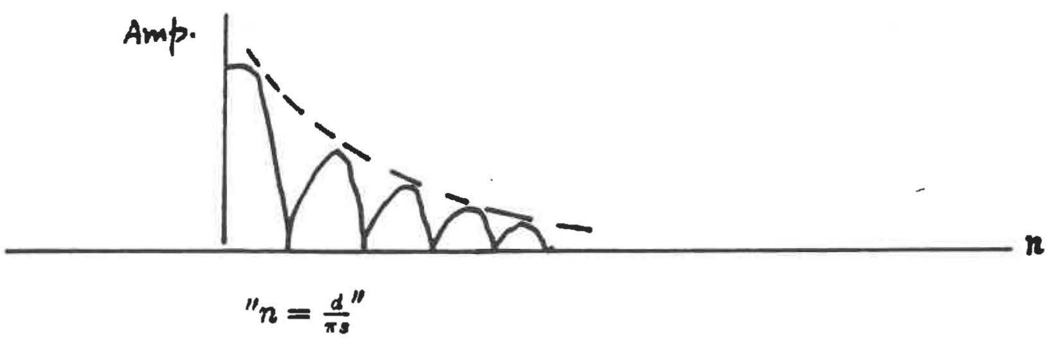


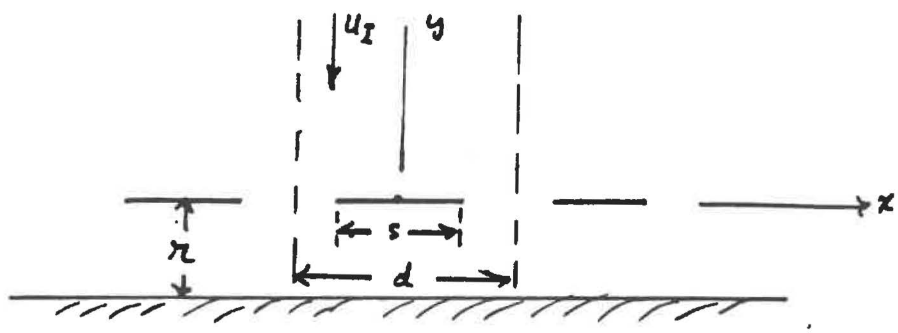

Fig.3.

Problem 2 Effect of a substrate.

The effect of a substrate a distance $r$ below (Fig.4.) the main grating is to reflect more of the light to $y > 0$. This can be represented by dipoles in the gaps at $y = 0$. This dipole strength is also assumed to be constant with an added phase shift $2r/c$ for the time it takes the light to return to the plane $y = 0$. The periodicity of this grating is the same as before and the result is basically the same. Let $D_1 = \text{const}$ be the dipole intensity of reflection on the substrate. Then

$$\widetilde{D}_n = \frac{D_o - D_1 e^{i\frac{2r}{c}}}{\pi} \; \frac{\sin\frac{n\pi s}{d}}{n}$$

and the previous formula applies with $\widetilde{u}_n$ modified.




substrate Fig.4.

Problem 3. Offset Grating

During the dynamic operation of the grating every third member of the grating is moved a distance $h$, for example, above the plane $y = 0$. Or it starts there and moves down into the plane under the mechanism of electrostatic attraction (or repulsion). (Fig. 5.)

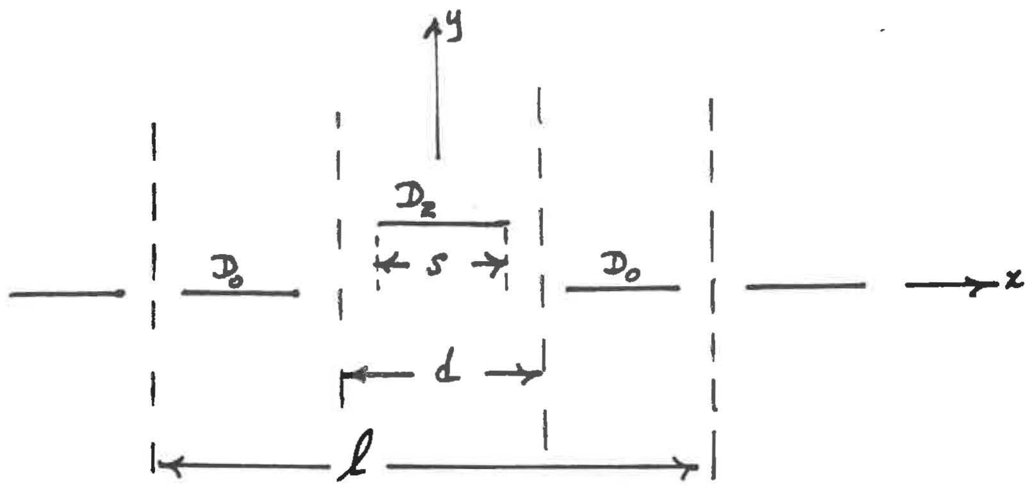



Fig.5.

The grating period is now $\ell = 3d$. The grating is again represented by dipoles of constant strength on the segments.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = D_0\,\delta'(y) + D_2\,\delta'(y-h)$$

For the parts in $y = 0$ we have the Fourier series (period $\ell$)

$$D_0 = \sum_{m=-\infty}^{\infty} \widetilde{D}_{0m}\, e^{i\beta_m x} \qquad \beta_m = \frac{m(2\pi)}{\ell} = \frac{m(2\pi)}{3d} = \frac{\alpha_m}{3}$$

$$\widetilde{D}_{0m} = \frac{D_0}{\ell}\int_{-d-\frac{s}{2}}^{-d+\frac{s}{2}} e^{i\beta_m x}\, dx \qquad + \frac{D_0}{\ell}\int_{d-\frac{s}{2}}^{d+\frac{s}{2}} e^{i\beta_m x}\, dx$$

$$\widetilde{D}_{0m} = \frac{2}{\pi}\, D_1 \qquad \frac{\cos\frac{m(2\pi)}{3}\sin\frac{m\pi s}{\ell}}{m.}$$

The part of the difraction pattern from these segments is given by

$$u_0(x,y) = \sum_{m=-\infty}^{\infty} \tilde{u}_{0m}(y)\, e^{i\beta_m x} = \frac{1}{2}\sum_{m=-\infty}^{\infty} \widetilde{D}_{0m}\, e^{i\left(\beta_m x + \sqrt{k^2-\beta_m^2}\, y\right)}$$

$\beta_m = k\sin\Theta_m$, $\sin\Theta_m = m\,\frac{\lambda}{\ell}$, basic grating formula, as before $[m] < \frac{\ell}{d} < \frac{3d}{\lambda}$. This primary result shows for the first order an angle (roughly) $\frac{1}{3}$ the deflection of the grating of period $d = \frac{1}{3}\ell$. Now more orders of diffraction will appear $\left(\frac{m\lambda}{\ell} < 1\right)$.

For the offset part of the grating

$$\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + k^2 u_2 = D_2\,\delta'(y-h)$$

where the Fourier series for $D_2$ is similar to that derived earlier

$$D_2 = \sum_{m=-\infty}^{\infty} \widetilde{D}_{2m}\, e^{i\beta_m x}$$

$$\widetilde{D}_{2m} = \frac{D_2}{\ell}\int_{-\frac{s}{2}}^{\frac{s}{2}} e^{-i\beta_m x}\, dx = \frac{D_2}{\pi}\ \frac{\sin\frac{m\pi s}{\ell}}{m}$$

The solution for $u_2$ is in the form

$$u_2(x,y) = \sum_{m=-\infty}^{\infty} \widetilde{u}_{2m}(y)\, e^{i\beta_m x}$$

$$\frac{d^2\widetilde{u}_{2m}}{dy^2} + (k^2 - \beta_m^2)\widetilde{u}_{2m} = \widetilde{D}_{2m}\, \delta'(y-h)$$

Thus

$$u_2 = \frac{1}{2}\sum_{m=-\infty}^{\infty} \widetilde{D}_{2m}\, e^{-i\sqrt{k^2-\beta_m^2}\,h} \quad e^{i\left(\beta_m x + \sqrt{k^2-\beta_m^2}\,y\right)}$$

where a phase shift $e^{-i\sqrt{k^2-\beta_m^2}\,h}$ has been introduced to account for the arrival of the incoming wave at $y = h$ sooner. The same orders appear here as in $u_0$. The total solution has

$$u(x,y) = u_0 + u_2 = \frac{D_0}{\pi}\sum_{m=-\infty}^{\infty}\frac{1}{m}\left(\cos\frac{m(2\pi)}{3} + \frac{1}{2}e^{ik(1+\cos\Theta_m)h}\right)\sin\frac{m\pi s}{\ell}$$

$$\cdot\, e^{ik(x\sin\Theta_m + y\cos\Theta_m)} \qquad \text{(B)}$$

(using $D_2 = D_0$ except for planeshift). As the elevation $h \to 0$ this formula approaches our first example in Problem 1.

Comments

(i) The main effect of introducing out of plane rulings is to change the periodicity. The first order of diffraction is the changed from $\sin\Theta_1 = \frac{\lambda}{d}$ to $\sin\Theta_1 = \frac{\lambda}{\ell} = \frac{\lambda}{3d}$. The amplitude of this first order is appreciable if $h$ is not small. It can be calculated roughly from (B).

(ii) It could be desirable to formulate this problem more rigorously and do some computations to get the real result.

(iii) The effect of the substrate can be included.

(iv) Other polarizations can be included.

(v) Non-normal incidence can be worked out.

(vi) The silicon elastic beam as one plate of a capacitor has a non-uniform force applied to it.