Filter Cleaning Using Gas Injection

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Abstract

A filter cleaning process using gas injection is considered. An estimate for the minimum mass flow rate out of the gas injector and the corresponding injector/filter geometry is found. The estimates are based on a similarity solution for a free turbulent jet. The minimum mass flow rate and geometry is worked out for a specific example.

1. Introduction

A typical surface filter requires a periodic cleaning to remove the filtrate build up and thus maintain good performance. There are many ways to accomplish the cleaning. One approach is to use a jet of clean gas of sufficient strength to induce a back flow through the filter to remove the cake of filtrate. The jet of clean gas comes from an injector located near the end of the filter and it is of interest to determine the geometry of the injector/filter system so that the filter can be cleaned using the smallest possible mass flow out of the injector. It is the aim of this report to obtain estimates of the amount of the jet flow that can be "captured" by the filter and thus determine the amount of mass flow required to clean the filter.

The basic injector/filter system is illustrated in Figure 1. The length of the cylindrical filter is L_f and its radius is R_f. In the forward flow, the filtrate builds up on the outside of the filter and the cleaned gas exits through the circular opening as shown. As the amount of the filtrate cake grows, the pressure required to maintain the forward flow increases and it desirable to clean the filter. For this purpose, an injector is located a distance L_i away from the filter and its radius is R_i. A "collector" with length L_c and radius R_c may also be included in the injector/filter system.

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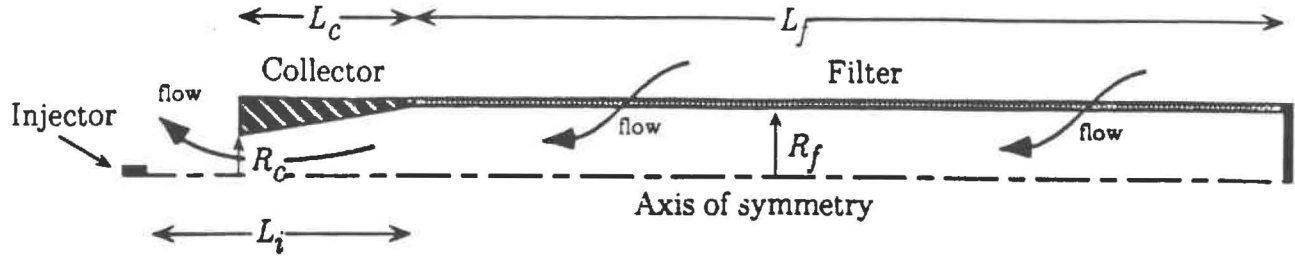
Figure 1. Injector/filter system.

The basic flow produced by the injector is taken to be a similarity solution for a free turbulent jet [1]. In this approximation, the injector provides a point source of momentum that spreads radially with axial distance. Mass is entrained by the jet to maintain momentum conservation. At the opening of the collector a certain mass of the jet is captured. A key assumption is that the mass captured at the collector opening is the total amount of mass in the free jet within a disc of radius R_c. It is assumed that the momentum contained in the captured flows acts a piston to increase the pressure in the filter. The condition that the pressure must increase by an amount Δp^* to remove the cake provides an estimate for the geometry and required mass flow out of the injector. The additional condition that there is a volume flow rate Q^* through the filter required to maintain Δp^* is also used.

In addition to estimates of the geometry and mass flow, comments are made concerning the mechanism by which the cake releases from the filter.

2. A Free Turbulent Jet

The Reynolds number of the gas flow from the injector is very large and it is assumed that R_i/R_c and $R_i/(L_i - L_c)$ are reasonably small and thus the opening of the collector sees a flow produced by the injector that is approximately a free axisymmetric turbulent jet driven by a point source of momentum. The details of this flow are given in [1] and the results needed for this report are outlined here.

The axial component of velocity u in the free jet is given by a similarity solution that takes the form

$$
\begin{aligned}
u(x,r) &= \frac{3}{8\pi}\frac{K}{\epsilon_0 x}\frac{1}{\left(1+\frac{1}{4}\eta^2\right)^2}, \\
\eta &= \frac{1}{4}\sqrt{\frac{3}{\pi}}\frac{\sqrt{K}}{\epsilon_0}\frac{r}{x},
\end{aligned}
\tag{1}
$$

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where x is axial distance from the point source, r is radial distance, ϵ_0 is the "virtual" kinematic viscosity and

$$K = 2\pi \int_0^\infty u^2 r \, dr$$

is a constant equal to the total kinematic momentum which determines the strength of the jet. From experiments it is found that

$$\frac{\epsilon_0}{\sqrt{K}} = 0.0161.$$

An effective source position $x = 0$ located behind the injector and a strength K of the jet are determined approximately by balancing both mass and momentum flux at the injector. The free jet solution in (1) is then used to obtain an estimate for the mass entering the collector.

3.   The Injector

For the discussion of the injector and the subsequent discussion of the flow at the collector, it is helpful to refer to Figure 2 which shows an enlarged view of the region between the injector and collector and gives some notation.
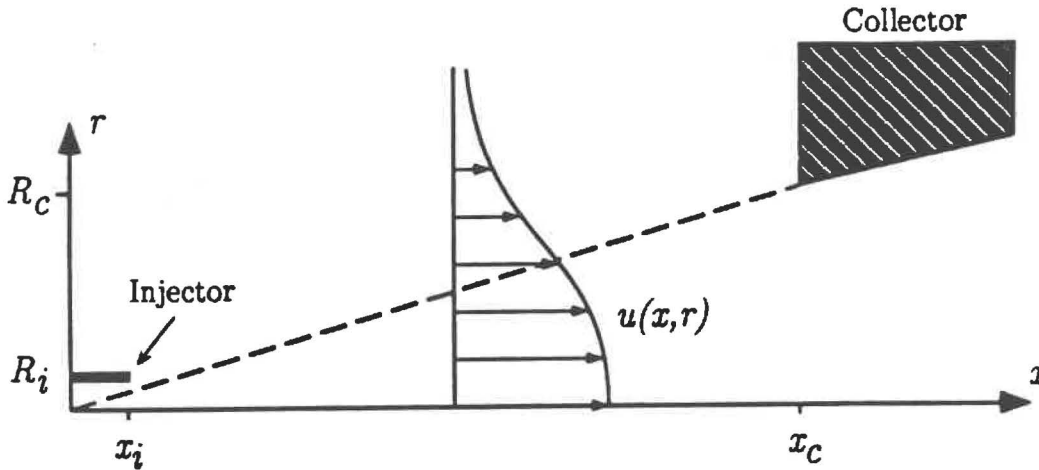

Figure 2. Injector/collector geometry and the effective free jet.

The injector is characterized by a mass flow rate M. Assuming a constant density ρ of the gas, the average velocity at the injector is $M/\rho\pi R_i^2$, where R_i is the radius of the injector, and this velocity is limited by the the speed of sound c. The flow produced by the injector is to be approximated by the free jet solution (1) whose strength K is related to M and whose

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source position $x = 0$ is located a distance x_i behind the injector. Balancing the kinematic momentum out of the injector with that of the jet gives

$$K = \frac{1}{\pi} \left(\frac{M}{\rho R_i} \right)^2 . \tag{2}$$

The distance x_i is found by assuming that the mass flow rate out of the injector is equal to the total mass flow rate in the jet at a distance x_i from the source. Using (1),

$$M = 2\pi\rho \int_0^\infty u(x_i, r) r \, dr = 8\pi\rho\epsilon_0 x_i,$$

which can be combined with (2) to give

$$\frac{x_i}{R_i} = \frac{1}{8\sqrt{\pi}} \frac{\sqrt{K}}{\epsilon_0} = 4.380. \tag{3}$$

4. The Collector

Consider the effective source of the free turbulent jet to be a distance x_c from the entrance to the collector which has radius R_c. In order to make a simple model of the complicated turbulent flow into the collector, we make the assumption that the mass flow rate into the collector is equal to that in the jet at a distance x_c from the source averaged within the radius R_c. Thus, the mass flow rate into the collector is given by

$$2\pi\rho \int_0^{R_c} u(x_c, r) r dr.$$

It is assumed further that the flow entering the collector spreads out in the first part of the filter to form an approximately uniform velocity u_c. The dynamic pressure $\frac{1}{2}\rho u_c^2$ of this uniform flow is then available to achieve the pressure increase Δp^* required to clean the filter. The remaining momentum within the turbulent jet flow is assumed to be dissipated in the process of redistributing the velocity.

According to our simple model, the two physical conditions that the flow has sufficient dynamic pressure to achieve Δp^* and that the volume flow rate is sufficient to sustain that pressure rise give

$$\frac{1}{2}\rho u_c^2 \geq \Delta p^* \tag{4}$$

and

$$\pi R_c^2 u_c \geq Q^*, \tag{5}$$

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where Q^* is the required volume flow rate. For a particular filter element, experiments are required to determine a value for Δp^* and the corresponding volume flow rate Q^* required to maintain that pressure increase. These two quantities are related by

$$\Delta p^* = \beta \left(\frac{Q^*}{A_f} \right),$$

where β is the permeability of the dirty filter and A_f is the surface area of the cylindrical filter.

Using the result in (1), we find that

$$u_c = \frac{2\pi \int_0^{R_c} u(x_c, r) r \, dr}{\pi R_c^2} = \frac{\sqrt{K}}{x_c} \cdot \frac{A}{1 + B^2 \left(\frac{R_c}{x_c} \right)^2},$$

where

$$A = \frac{3}{8\pi} \frac{\sqrt{K}}{\epsilon_0} = 7.414 \qquad \text{and} \qquad B = \sqrt{\frac{\pi}{3}} A = 8.130.$$

Thus the two conditions, (4) and (5), become

$$\frac{\rho K}{2 x_c^2} \frac{A^2}{\left(1 + B^2 \left(\frac{R_c}{x_c} \right)^2 \right)^2} \geq \Delta p^* \tag{6}$$

and

$$\frac{\pi R_c^2 \sqrt{K}}{x_c} \frac{A}{1 + B^2 \left(\frac{R_c}{x_c} \right)^2} \geq Q^*. \tag{7}$$

5.   Design Criteria

The conditions in (6) and (7) provide an estimate for the minimum value of M required to clean the filter and the corresponding values of the x_c and R_c for the collector. For values of M greater than the minimum, the conditions would give a range of values for x_c and R_c.

Figure 3 shows the regions given by (6) and (7) for a fixed value of $\sqrt{K}$ which, by (2), is proportional to M. As M increases, $\sqrt{K}$ increases and the two regions grow until they touch and then overlap. The value of M when the regions first touch is the minimum value that satisfies both conditions. To find the minimum, take (6) and (7) to be equalities and solve for R_c^2 in both. These can be written in the form

$$\left(\frac{2\pi}{3} \frac{\Delta p^*}{\rho K} \right) R_c^2 = x_c' - x_c'^2 = \frac{x_c'^2}{\alpha x_c' - 1}, \tag{8}$$

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where

$$x_c' = \frac{x_c}{A}\sqrt{\frac{2\Delta p^*}{\rho K}} \qquad \text{and} \qquad \alpha = \frac{3}{\sqrt{2}}\frac{K}{Q^*\sqrt{\Delta p^*/\rho}}.$$

The second equality in (8) reduces to the quadratic

$$\alpha x_c'^{\,2} - \alpha x_c' + 1 = 0,$$

whose solutions give the intersections of the boundaries of the two regions. The boundaries first touch when $\alpha = 4$ which gives the minimum K. We find that

$$K_{\min} = \frac{4\sqrt{2}}{3}Q^*\sqrt{\frac{\Delta p^*}{\rho}}. \tag{9}$$

At this minimum, $x_c' = 1/2$ so that

$$x_c = \frac{A}{2}\sqrt{\frac{\rho K_{\min}}{2\Delta p^*}} \qquad \text{and} \qquad R_c = \frac{x_c}{B}. \tag{10}$$
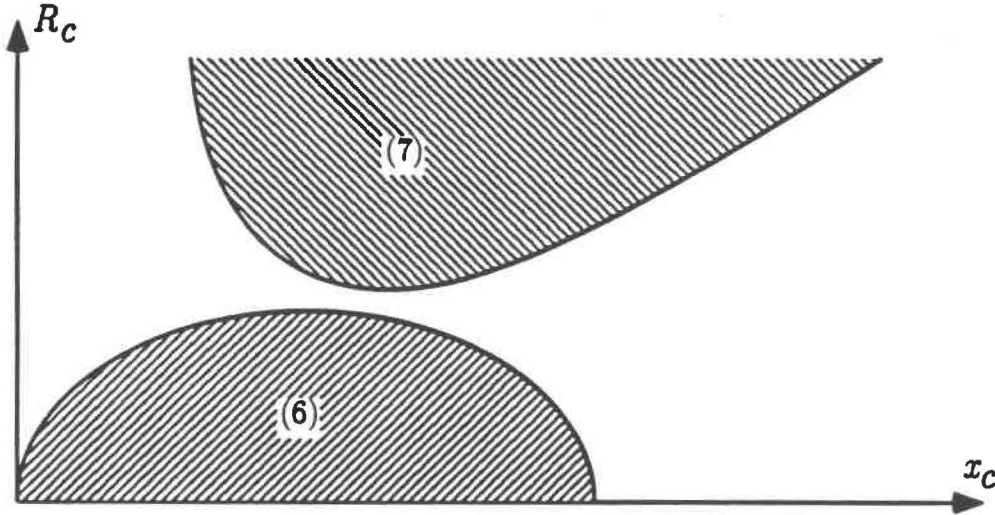

Figure 3. Admissible regions given by (6) and (7).

6. An Example

In this section, we calculate the minimum mass flow rate out of the injector and the geometry of the injector and collector for a specific example. The gas properties for this example are taken to be

$$\rho = 4.65\,\frac{\text{kg}}{\text{m}^3}, \qquad c = 672\,\frac{\text{m}}{\text{s}}.$$

The geometrical parameters of the cylindrical filter are

$$L_f = 1.50\,\text{m}, \qquad R_f = 0.03\,\text{m}, \qquad A_f = 0.283\,\text{m}^2.$$

The filter permeability takes the values

$$180.\,\frac{\text{Pa}}{\text{m/s}} \geq \beta \geq 3.4 \times 10^5\,\frac{\text{Pa}}{\text{m/s}},$$

where the higher values are taken when the filter is dirty. Finally, it has been found by experiment that the pressure excess needed to clean this filter is

$$\Delta p^* = 4.9 \times 10^4\,\text{Pa}$$

Using the parameters given above, the first task is to calculate K_{min} using (9). To do this, we first calculate Q^*:

$$Q^* = \frac{\Delta p^* A_f}{\beta} = 0.0407\,\frac{\text{m}^3}{\text{s}}.$$

Thus,

$$K_{\text{min}} = 7.89\,\frac{\text{m}^4}{\text{s}^2}.$$

In order to calculate the minimum mass flow rate out of the injector, we assume that the flow speed at the exit of the injector is sonic. Thus,

$$M_{\text{min}} = \frac{\rho K_{\text{min}}}{c} = 0.0546\,\frac{\text{kg}}{\text{s}}.$$

The geometry of the injector follows from (2) and (3) and the geometry of the collector follows from (10). We find that

$$R_i = 0.00236\,\text{m}, \qquad x_i = 0.0103\,\text{m}, \qquad R_c = 0.00882\,\text{m}, \qquad x_c = 0.0717\,\text{m}.$$

For this example, the distance between the injector and the collector is $x_c - x_i = 0.0614\,\text{m}$.

7. Cake Release

In order to extrapolate $\Delta p*$ and Q^* from values determined experimentally for various cake layers, it is useful to explore possible mechanisms for cake removal. The cake layers can be of many kinds. For a simple non-adhering cake, $\Delta p*$ is minimal since any reverse flow through the filter surface will dislodge the layer. The main restriction in this case is that the flow be sufficient to ensure that the removed cake has a chance to partially settle so that it is not immediately put back onto the surface when forward flow resumes.

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For more adhesive cakes there will be a critical pressure where the cake will be removed. The adhesive cake layer may act as a thin layer of elastic material. If the cake is not well-adhered to the filter surface then the dominant failure mechanism is due to excessive hoop stress developed by the pressure drop through the cake layer. This would indicate that the cake layer will release by a crack propagating quickly up the cake layer (an "unzipping action") resulting in the layer being removed in a single large sheet. Such a model would indicate that the excess pressure drop required to remove the layer would decrease as the thickness of the layer was reduced. In practice the cake is reasonably well adhered to the filter and therefore, although the observed failures may be by unzipping, such a failure would not occur until the cake has become detached from the surface by a tensile failure. Such a failure will be relatively insensitive to layer thickness and primarily dependent on the excess pressure applied. Such a model would indicate that the excess pressure drop required to remove the layer was independent of the layer thickness. A small experimental study might determine which of the two modes outlined is most important in practice.

Acknowledgements

References

[1] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York, 1979.