Two-phase flow in a thin, porous substance.
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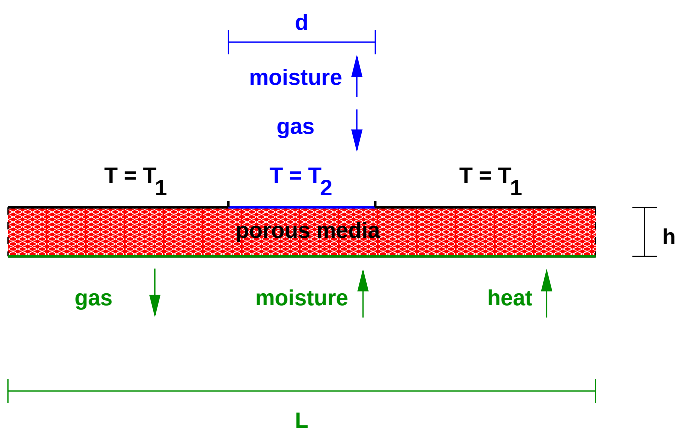
1 Physical configuration

The challenge is to describe the steady-state and dynamic transport of gas, u, water, θ, and heat, q, in a thin porous substance. The water is in liquid and vapor phase. The porous media itself has properties including an intrinsic permeability K and porosity ϕ. The intrinsic permeability has yet to be characterized experimentally. The problem is periodic in the horizontal direction. The lower boundary has a specified flux of heat, water and gas. The upper boundary has two zones. One zone is impermeable and held at a fixed temperature, T_1. The second zone is held at a fixed temperature T_2, and permits evaporation and gas transport. The gas transport in from the top and out through the bottom are necessarily in balance for the steady state problem.



Typical parameter values are roughly $0.7 \leq \phi \leq 0.8$, $h \approx 0.2$ mm, $L \approx 2$ mm, $d \approx 1$ mm. The intrinsic permeability of the substance has not be characterized, but this will be discussed in more detail at the workshop. The dry, through-plane permeability is thought to be about 10^{-12} m^2.

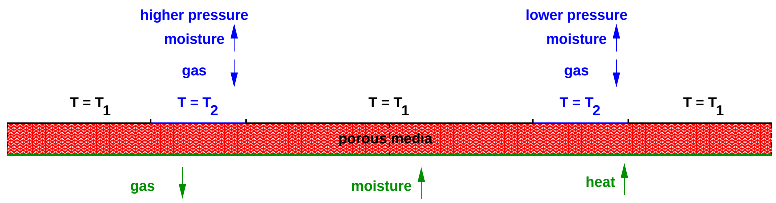
2 The problems

1. If the gas, moisture and heat flux are spatially uniform along the bottom surface, what is the distribution of gas, moisture (both vapor and liquid) and heat in the porous media?

2. Suppose the gas, moisture and heat flux vary linearly with the gas concentration at the lower boundary, that is

$$\frac{\partial u}{\partial y} - c_1 u = 0,$$
$$\frac{\partial \theta}{\partial y} - c_2 u = 0,$$
$$\frac{\partial q}{\partial y} - c_3 u = 0,$$

on the lower boundary where $c_i > 0$, what is the distribution of gas, moisture (both vapor and liquid) and heat in the porous media?



3. If the configuration has two input zones on the upper boundary and the gas pressure differs between the two, what is the distribution of gas, moisture (both vapor and liquid) and heat in the porous media? Can these solutions be extended for three zones or more?

3 Background material

Some insight may be gained from geophysical work on rainfall infiltration though these flows are driven by a gravitational potential rather than thermal and chemical gradients, and imposed pressure fields. In this problem, the porous media has higher porosity and smaller pore sizes than geophysical materials, and permeability dependence on moisture content may be very different.

References

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