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# Performance Across Different Areas of Mathematical Cognition in Children With Learning Difficulties

Laurie B. Hanich, Nancy C. Jordan, David Kaplan, and Jeanine Dick
University of Delaware

The performance of 210 2nd graders in different areas of mathematical cognition was examined. Children were divided into 4 achievement groups: children with difficulties in mathematics but not in reading (MD-only), children with difficulties in both mathematics and reading (MD/RD), children with difficulties in reading but not in mathematics, and children with normal achievement. Although both MD groups performed worse than normally achieving groups in most areas of mathematical cognition, the MD-only group showed an advantage over the MD/RD group in exact calculation of arithmetic combinations and in problem solving. The 2 groups did not differ in approximate arithmetic and understanding of place value and written computation. Children with MD-only seem to be superior to children with MD/RD in areas that may be mediated by language but not in ones that rely on numerical magnitudes, visuospatial processing, and automaticity.

Educational and cognitive psychologists are beginning to devote serious attention to young children with mathematics difficulties (MD), a population that previously had been understudied (Ginsburg, 1997). Models of mathematical cognition in normally developing children have influenced the current work in mathematics difficulties, especially in the areas of counting knowledge (e.g., Briars & Siegler, 1984; Fuson, 1988; Geary, Bow-Thomas, & Yao, 1992; Gelman & Gallistel, 1978), arithmetic operations (e.g., Huttenlocher, Jordan, & Levine, 1994; Jordan, Levine, & Huttenlocher, 1995; Levine, Jordan, & Huttenlocher, 1992), problem solving (e.g., Riley & Greeno, 1988), and strategy use (e.g., Siegler & Jenkins, 1989). These quantitative abilities, which develop early in life, lay important foundations for higher-level mathematical competence. Avoidance of mathematics is no longer inconsequential in our technology-oriented culture. Deficiencies in mathematical competence can seriously limit a student's educational opportunities (Rivera-Batiz, 1992).

Two major research issues have emerged from recent investigations of cognitive processes in children with MD. The first issue involves participant-identification procedures. In many studies, children with MD are defined as a single group of low achievers (e.g., Geary, 1990; Ostad, 1999; Russell & Ginsburg, 1984); children with MD alone are not differentiated from children with both MD and reading disabilities (RD). However, children with MD who are good readers show a different pattern of cognitive deficits than children with MD who are poor readers (Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Hamson, 1999; Jordan, Blanteno, & Uberti, in press; Jordan & Hanich, 2000; Rourke & Conway, 1997), with the former having circumscribed deficits and the latter having more general ones. A second, related issue involves the domains of mathematical cognition that are assessed. Much of the research on children with MD is narrowly focused, emphasizing only one area of mathematical competence. Children's computational skills, in particular, have received considerable attention, whereas problem solving and numerical understanding have received comparatively little attention (Jordan & Hanich, 2000). Because different aspects of mathematics involve different cognitive abilities (Carroll, 1996; Geary et al., 2000), mathematics difficulties may be uneven across domains (Ginsburg, 1997). For example, some children might have relative weaknesses in fact retrieval, even though they understand counting principles and mathematical concepts, whereas others might have relatively strong computational skills despite a weak understanding of concepts (Jordan & Hanich, 2000; Jordan & Montani, 1997; Russell & Ginsburg, 1984). Competencies across and within different areas of mathematics should be studied in children with MD.

The present study is the first piece of a multiyear longitudinal project on the development of mathematical competencies in young children. It addresses the aforementioned issues by examining second graders with difficulties in mathematics but not in reading (MD-only) and second graders with difficulties in mathematics as well as in reading (MD/RD). For comparison, we included a group of children with difficulties in reading but not in mathematics (RD-only) and a group with normal achievement in reading and mathematics (NA). We assessed areas of mathematical cognition that are directly related to the teaching of mathematics (as opposed to more general cognitive competencies), including basic calculation, approximate arithmetic, problem solving, place value, and written multidigit computation. In the following section, we provide a research-based rationale for including tasks in each area.

Laurie B. Hanich, Nancy C. Jordan, David Kaplan, and Jeanine Dick, School of Education, University of Delaware.

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Correspondence concerning this article should be addressed to Nancy C. Jordan, School of Education, University of Delaware, Newark, Delaware 19716. Electronic mail may be sent to njordan@udel.edu.

## Basic Calculation

Arithmetic combinations involve problems such as "How much is 3 and 4?" or "How much is 7 take away 3?" Skilled performance in simple arithmetic develops gradually during early childhood (e.g., Jordan, Huttenlocher, & Levine, 1992; Jordan, Levine, & Huttenlocher, 1994; Levine et al., 1992; Siegler, 1991). In preschool and kindergarten, many children solve arithmetic combinations by making rough estimates or by guessing. Children gradually learn to represent the problems with their fingers or other physical referents and to use these referents to count both addends, in the case of addition (counting all), or to separate a subtrahend from the minuend, in the case of subtraction (separating from). Children may also use visualization or direct retrieval on familiar combinations involving small numerosities (e.g., $1 + 1$ or $2 - 1$). By second grade, children develop efficient counting strategies (e.g., for addition problems, they use a counting-on procedure, which involves stating the larger addend and then counting upward the number of times equal to the value of the smaller addend). Moreover, they begin to use calculation "short-cuts" (Baroody, 1999; Dowker, 1998; Russell & Ginsburg, 1984), such as deriving answers from known number facts (e.g., the doubles plus one pattern, $2 + 2 = 4$, so $2 + 3 = 5$) and applying basic principles (e.g., the commutativity principle, $2 + 1 = 3$, so $1 + 2 = 3$, and the inversion principle, $3 + 1 = 4$, so $4 - 1 = 3$). Knowledge of calculation principles reflects an understanding of the relationships within and between arithmetic operations (Jordan et al., in press). By the end of third grade, the majority of children retrieve or construct answers by deriving answers from known arithmetic combinations with minimal cognitive effort.

Children with MD have persistent weaknesses in automatic retrieval of number facts (e.g., Geary, 1990; Geary, Brown, & Samaranayake, 1991; Ostad, 1997, 1999; Russell & Ginsburg, 1984). Although both children with MD-only and children with MD/RD have difficulties retrieving facts quickly, children with MD-only appear to use counting strategies more effectively and have a better grasp of counting principles than do children with MD/RD (Geary et al., 1999; Jordan et al., in press; Jordan & Hanich, 2000; Jordan & Montani, 1997).

In the present study, we included three basic calculation tasks: (a) an arithmetic-combinations task, which allowed us to observe children's calculation accuracy as well as their strategy use (e.g., how often children count on their fingers) (Jordan & Hanich, 2000)—children were asked to use any strategy that would help them get the correct answer; (b) a principles task that examined children's knowledge of the commutativity and inversion principles and the doubles plus one pattern; and (c) a "forced retrieval" task (Jordan & Montani, 1997), which required children to retrieve answers to arithmetic combinations automatically. We predicted that the performance of children with MD-only and children with MD/RD would be more differentiated on the untimed arithmetic-combinations task and on the principles task than on the forced (rapid) retrieval task. Rapid retrieval deficits may be present in the RD-only population as well (Geary et al., 2000).

## Approximate Arithmetic

Approximate arithmetic requires individuals to estimate results, in contrast to exact arithmetic, which requires them to give an exact answer. When solving approximate arithmetic problems (e.g., $9 + 8 = 20$ or 30), individuals must form a "mental number line" to manipulate and estimate quantities (Dehaene & Cohen, 1991). Approximate arithmetic involves visuospatial abilities that seem to be independent of language (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Exact calculation (e.g., the ability to answer arithmetic combinations), on the other hand, appears to be acquired in a language-specific format. Because Dehaene et al.'s (1999) work was conducted with adults, however, it is not known whether children's performance can be differentiated on approximate (estimation) and exact arithmetic tasks.

In the present study, we adapted Deheane et al.'s (1999) approximate arithmetic task for use with children. To the extent that the deficits of MD-only and MD/RD children are related to spatial abilities, both groups of children should show deficits on an approximate arithmetic task (relative to the performance of children without MD). We also expected that children with MD-only would perform about as poorly as children with MD/RD on an approximate calculation task but would have an advantage over MD/RD children on an exact calculation task (i.e., arithmetic combinations), where performance is facilitated by language (e.g., verbal counting).

## Problem Solving

In elementary school, children learn to solve mathematics story problems that involve basic arithmetic operations but vary in semantic complexity (Riley, Greeno, & Heller, 1983). These problems are referred to as (a) *change* (e.g., "Nina had 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now?"); (b) *equalize* (e.g., "Claire has 4 pennies. Ben has 9 pennies. How many pennies must Claire get to have as many as Ben?"); (c) *combine* (e.g., "Maria and Kevin have 5 pennies altogether. Maria has 3 pennies. How many pennies does Kevin have?"); and (d) *compare* (e.g., "Anna has 7 pennies. She has 2 pennies less than Larry does. How many pennies does Larry have?"). Skill in solving these types of problems increases gradually in elementary school, with change and combine problems being the easiest and equalize and compare the hardest (Riley & Greeno, 1988).

Story problem solving is an area of significant weakness for children with MD. Ostad (1998) compared the performance of children with MD on a set of change, equalize, combine, and compare problems. Children with MD performed worse than normally achieving peers on all problem types, a finding that held at three grade levels (i.e., second, fourth, and sixth grades). However, Ostad (1998) did not differentiate between children with MD-only and children with MD/RD. Because children in the MD group performed significantly below the mean on tests of verbal and spatial abilities, the reported achievement group differences on story problems could have been related to general IQ differences.

Using story problems involving a change, Jordan and Montani (1997) found that third-grade children with MD-only performed better than their peers with MD/RD and as well as their normally achieving peers when the task was untimed but not when it was timed. Children with MD-only may have deficits associated with problem-solving speed rather than with basic problem comprehension. In a subsequent study with second graders, Jordan and

Hanich (2000) used more complex story problems (e.g., equalize and compare problems) than those used by Jordan and Montani (1997). Children with MD-only had an advantage over children with MD/RD in an untimed condition but they performed worse than normally achieving children. Higher order problem solving is an area of weakness for both groups of children with MD (Jordan & Hanich, 2000). To replicate or expand upon the results of previous studies, children in the present investigation were given a set of story problems involving the change, combine, equalize, and compare categories.

## Understanding of Place Value and Skill in Written Multidigit Computation

In a longitudinal investigation, Hiebert and Wearne (1996) found a close connection between children's understanding of multidigit numbers and their written computational skills. Children who developed the earliest understanding of place value and base-10 concepts in first grade performed at the highest level in written computation in third grade.

Jordan and Hanich (2000) examined understanding of place value and skill in written computation in second-grade children with MD. Although children with MD/RD were pervasively weak, the majority of children with MD-only and children without MD appeared to understand multidigit numbers on a chip-trading activity. (For example, on a two-digit task, the child was told that yellow chips are worth 1 point and red chips are worth 10 points. He was then shown a card with the number 32 written on it and asked to show the same amount with the chips.) However, as the investigators point out, the nature of the task may have overestimated many children's understanding. That is, through rote learning in school, the child may have known that the 3 in 32 represents three red chips without understanding that this represents 30. Hence, in the present investigation, we adapted tasks designed by mathematics educators (Hiebert & Wearne, 1996; Kamii, 1989; Ross, 1989) to tap different levels of understanding of and skill with place value. These tasks included counting and number identification, positional knowledge (i.e., identifying numbers in the ones, tens, and hundred places), and digit correspondence (i.e., showing the meaning of numbers with standard and nonstandard place-value partitioning). The digit correspondence task was designed specifically to examine children's understanding of two-digit numbers (Ross, 1989). To assess multidigit written computation skills, we also asked children to solve paper-and-pencil calculations with and without regrouping.

In sum, in the present study, we investigated mathematical competencies in children with different patterns of achievement in mathematics and reading. Our goal was to examine children's performance in four areas of mathematics that are relevant to learning in early elementary school. Our interest was to determine whether the tasks would differentiate performance among the various achievement groups, particularly between children with MD-only and children with MD/RD. To the greatest extent possible, ethnicity and socioeconomic status (SES) were balanced within achievement groups. In many investigations of learning difficulties (e.g., Jordan & Hanich, 2000), achievement group and ethnicity–SES have been confounded (i.e., children in difficulty groups tend to be minorities from low-income families, whereas children in normally achieving groups tend to be White and middle class).

## Method

### Participants

Participants were 210 second-grade children with different patterns of achievement. Fifty-three children had difficulties in mathematics but not in reading (MD-only); 52 children had difficulties in mathematics as well as in reading (MD/RD); 50 children had difficulties in reading but not in mathematics (RD-only); and 55 children had normal achievement both in reading and mathematics (NA).

*Participant selection procedure.* Informed consent letters were sent to the parents or guardians of all second-grade children ($n = 919$) in six schools in the same school district in Northern Delaware. We received permission to test 72% of the children ($n = 664$). Permission rates were comparable across schools. From this pool, we were able to screen 643 children.

Reading and mathematics achievement were assessed in the fall of second grade with the Woodcock-Johnson Tests of Educational Achievement, Form A (WJTEA; Woodcock & Johnson, 1990). The WJTEA is a norm-referenced, individually administered assessment battery. The reading portion of the test includes letter–word identification and passage comprehension subtests, and the mathematics portion includes calculation and applied problems subtests. Reliability and validity of the reading and mathematics portions of the WJTEA are well established (Salvia & Ysseldyke, 1998).

Children with mathematics composite scores at or below the $35^{th}$ percentile were classified as MD and children with reading composite scores or letter–word identification scores at or below the $35^{th}$ percentile were classified as RD. The $35^{th}$ percentile cutoff is somewhat higher than is typically used in research in learning disabilities (i.e., $25^{th}$ percentile), but this cutoff was necessary to ensure adequate sample sizes in young children (Geary et al., 2000). Moreover, we were interested in second-grade children who may be at risk for school-related learning disabilities. Normal achievement was defined as scoring at or above the $40^{th}$ percentile on the mathematics or the reading composite. All children in the MD-only and RD-only groups had at least a 10-point discrepancy between mathematics and reading achievement. (The average mathematics and reading discrepancy for the MD-only group was 49 percentile points and for the RD-only group was 34 percentile points). The mean reading and mathematics percentile scores on the WJTEA are presented in Table 1. We selected children in the NA group by matching the reading levels of participants as closely as possible to children in the MD-only group and their mathematics levels to children in the RD-only group. NA children were selected from the same classrooms from which children with difficulties were selected.

Table 1 also includes information about SES, gender, and ethnicity for children in each achievement group. The four achievement groups were balanced by ethnicity and gender to the greatest extent possible. Within the school district, approximately 60% of the children are Caucasian, 33% are African American, 4% are Hispanic, and 3% are Asian. All of the participants attended general education classes. Thirteen percent ($n = 27$) of the children in the sample were identified by the school district as needing special-education services (1 MD-only child, 18 MD/RD children, and 8 RD-only children).

*Instructional programs.* For mathematics instruction, teachers of all children in the school district used the textbook *Math* (Scott Foresman–Addison Wesley, 1998). Supplemental materials developed by the Technical Education Research Centers and the National Science Foundation also were used. Most teachers in the district reported on a questionnaire that they encouraged children to use their fingers to help them solve arithmetic combinations. Many also reported that they used timed arith-

Table 1
*Descriptive Information for Participants by Achievement Group*

| Achievement group | $n$ | Male/Female | Percentage ethnic minority[a] | Percentage low SES[b] | Reading composite percentile scores | Letter–word identification percentile scores | Mathematics percentile scores |
|---|---|---|---|---|---|---|---|
| MD-only | 53 | 24/29 | 58 | 42 | $71.87_a$ (14.60) | $67.29_a$ (17.91) | $22.94_c$ (9.30) |
| MD/RD | 52 | 28/24 | 56 | 56 | $23.38_b$ (13.37) | $19.75_b$ (11.42) | $21.37_c$ (10.37) |
| RD-only | 50 | 32/18 | 56 | 52 | $27.38_b$ (10.04) | $23.62_b$ (9.78) | $61.34_b$ (15.81) |
| NA | 55 | 28/27 | 45 | 44 | $72.64_a$ (13.22) | $64.53_a$ (16.74) | $68.02_a$ (12.36) |

*Note.* Standard deviations are shown in parentheses. Means in the same column that do not share a subscript differ significantly at $p < .05$ in the Tukey honestly significant difference comparison. MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.
[a] Within each achievement group, children identified as ethnic minority were primarily African American (>80% for each achievement group). [b] Low socioeconomic status (SES) was determined by eligibility for the subsidized lunch program at school.

metic activities. The reading program was less prescribed in the school district. Most of the teachers of children who participated in our study claimed to use a balanced approach, one that combines multiple reading components (e.g., word attack, comprehension, etc.).

## *Materials and Procedures*

Children were assessed individually at school by one of four experimenters. The experimental mathematics tasks were given during January and February of second grade. The testing session lasted approximately 45 min. Before the assessment, the experimenters were fully trained in testing and strategy observation procedures with a videotaped test session and subsequent pilot-test sessions with 12 second graders. During the pilot testing, the experimenters worked in pairs, with one giving the tasks and the other observing. Every experimenter had the opportunity to work with each of the other three experimenters. Procedural questions and disagreements were noted and later resolved through discussion.

Each child was given seven mathematics tasks presented in the same order. The tasks, in order of presentation, included: (a) exact calculation of arithmetic combinations, (b) story problems, (c) approximate arithmetic, (d) place value, (e) calculation principles, (f) forced retrieval of number facts, and (g) written computation. Exact calculation of number facts was given first so children's selection of strategies would not be biased by subsequent calculation tasks. The order of the remaining tasks was selected to provide variety and to sustain children's interest. To make the required operations more salient and to optimize performance, we separated addition and subtraction items on exact calculation of arithmetic combinations, approximate arithmetic, forced retrieval of arithmetic combinations, and written computation.

*Exact calculation of arithmetic combinations.* Four addition and four subtraction arithmetic combinations were presented to each child (i.e., 9 + 8; 3 + 6; 5 + 6; 8 + 7; 9 − 3; 17 − 9; 11 − 5; 15 − 8). In our previous work using mixed addition and subtraction items, some children added on all problems regardless of the operation. The problems were presented both orally and visually (in a horizontal format) to children. The experimenter read the addition problems as "How much is A plus B?" and the subtraction problems as "How much is A minus B?" The written version of the problems was shown at the same time the problems were read. Children were told to use any method they wanted to figure out the answer and to give an oral response as soon as they knew the answer. (Because we were especially interested in observing how often children spontaneously use their fingers, we did not provide counters.) Immediately after reading the problem, the experimenter started timing the child with a stopwatch. As soon as the child began to state a solution to the problem, the experimenter stopped timing. If the child gave an answer but then wanted to think some more and change it (in most cases the child immediately stated "no" after giving the initial answer), the stopwatch was restarted or the experimenter made a best estimate of how many additional seconds the child took. Response times for each number combination were recorded. The experimenters agreed 96% on recording response times for a sample set of trials.

On each arithmetic combination, the experimenter observed the child for indications of strategy use (e.g., counting verbally or with fingers, retrieval, etc.) and recorded exactly what the child did on the score sheet. Immediately after the child gave an answer, the experimenter asked the child how he or she figured out the answer. The child's responses were recorded verbatim. Statements such as "I memorized it," "I learned it," and "I remembered" were considered to be indications of fact retrieval. We found that the child and the experimenter agreed on 97% of the trials, consistent with previous research (Geary et al., 2000; Siegler, 1987). For disagreements, the experimenter's observation was used if the strategy she observed was obvious (e.g., finger counting). If the experimenter's observations were ambiguous or if the experimenter did not observe any strategies, then the child's response was used (Geary et al., 2000).

After the testing, children's calculation strategies were classified according to the following categories by one of three data coders: finger counting–physical referents (the child used fingers to calculate); verbal counting (the child demonstrated counting behaviors without the aid of fingers or reported using verbal counting); derived fact (the child showed or reported having derived an answer from a known number fact, such as 5 + 5 = 10, so 5 + 6 = 11); automatic retrieval (the child did not show or report an observable strategy and answered in 3 s or less); and delayed retrieval (the child did not show or report an observable strategy and took longer than 3 s to answer). Similar classification categories have been described and validated by DeCorte and Verschaffel (1987), Geary et al. (2000), Jordan and Hanich (2000), and Ostad (1999). To ensure interrater reliability, eight randomly selected protocols were independently coded by each data coder. There was 98% agreement on the strategy classifications.

*Story problems.* Ten story problems ranging from conceptually simple to conceptually complex were presented orally to children (Carpenter & Moser, 1984; Riley & Greeno, 1988; Riley et al., 1983). Four types of story problems were included: change problems (unknown result, unknown change, and unknown start), combine problems, compare problems, and equalize problems. Story problems were presented to children in a fixed random order, with the exception of Items 1 and 2. These two items were conceptually simple change problems. The story problems, grouped by category, are shown in Table 2. To allow children to focus on problem solving rather than on calculation, all problems involved sums and minuends of the number 9 or lower.

Before reading the individual problems, the experimenter gave the children a container of plastic pennies and told them to use whatever strategy they wanted to get the correct answer. A written version of the problem was shown as the problem was read. Children were told to wait until the experimenter had finished reading the problem before giving a response. The timing procedures for children's responses were the same as

Table 2
*Story Problems by Category*

| Change |
|---|
| Nina had 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now? (unknown result) |
| Jen had 7 pennies. Then she gave some pennies to Joe. Now Jen has 2 pennies. How many pennies did she give to Joe? (unknown change) |
| Karen had some pennies. Then Matt gave her 4 more pennies. Now Karen has 6 pennies. How many pennies did she have to start with? (unknown start) |
| **Combine** |
| Emily has 3 pennies. John has 6 pennies. How many pennies do they have altogether? |
| Maria and Kevin have 8 pennies altogether. Maria has 3 pennies. How many pennies does Kevin have? |
| **Compare** |
| Dennis has 7 pennies. Molly has 5 pennies. How many pennies does Dennis have more than Molly? |
| Janet has 3 pennies. Andy has 5 more pennies than Janet. How many pennies does Andy have? |
| Anna has 7 pennies. She has 2 pennies less than Larry. How many pennies does Larry have? |
| **Equalize** |
| Claire has 4 pennies. Ben has 9 pennies. How many pennies does Claire need to get to have as many as Ben? |
| Alex has 8 pennies. Kris has 6 pennies. How many pennies does Alex need to give away to have as many pennies as Kris? |

those used for arithmetic combinations. The experimenters agreed 98% on recording response times for a sample set of trials.

The observation procedures for classifying children's problem-solving strategies were the same as those used on the exact calculation of arithmetic combinations. However, penny counting was added to the classification scheme. Children were not queried on how they reached solutions because of time constraints. Thus, strategies were classified only on the basis of experimenter observations. The derived-fact strategy category was eliminated because this strategy was never observed.

*Approximate arithmetic.* The approximate arithmetic task was based on materials and procedures used by Dehaene et al. (1999). Ten addition and 10 subtraction problems were presented to children along with two proposed answers (e.g., 4 + 5 = 10 or 20; 16 − 7 = 4 or 8). Items and their order of presentation are shown in Table 3. As each item was read to the child, it was also displayed in a written format. The children were told to respond right away and that they should not calculate the exact answer to the problem. Rather, they should choose the number that is closest to the actual answer. Both of the answers to the problem were false, but one of the answers was within a few units of the actual answer, whereas the other answer was more distant. To prevent children from calculating, a 5-s time limit for each problem was imposed. Immediately after reading each problem, the experimenter began timing. If the child did not respond within 5 s, a response of "no answer" was recorded and the problem was scored as incorrect. Before moving to the next problem, the child was encouraged to respond quickly. Each child was presented with two practice problems.

*Place value.* Three different place-value activities were used: (a) counting and number identification, (b) positional knowledge, and (c) digit correspondence. The activities were adapted from Hiebert and Wearne (1996), Kamii (1989), and Ross (1989). There were 12 items on the place-value task.

In the counting activity, the child was given 16 colored chips and asked to count the chips. If the child erred in counting, the experimenter counted the chips aloud to ensure that the child understood there were 16 chips altogether (see the description of the digit correspondence activity, below, for a follow-up task). In the three number-identification tasks, the experimenter showed the child a card with a number on it (e.g., 16, 37, 415) and asked the child to read the number aloud.

In the positional knowledge activity, after the child read the number 37 aloud (as described above), the child was then asked which number was in the tens place and which number was in the ones place. A similar request was made for the number 415, with the child questioned as to which numbers were in the hundreds, ones, and tens places. The items were scored using a pass–fail criterion in which all of the digit places needed to be identified correctly for the problem to be scored as correct.

The first digit correspondence activity followed the number-identification task for the number 16. Using the same card that was shown to the child, the experimenter circled the 6 in the number 16 with the eraser of a pencil and asked the child to use the chips to show what that part stands for in the number 16. (The correct answer required the child to show 6 chips.) The experimenter then circled the 1 on the 16 card and asked the child to use the chips to show what that part stands for in the number 16. (The correct answer required the child to show 10 chips.)

The next digit correspondence activity was a paper-and-pencil task that examines children's understanding of two-digit numbers. It is based on activities developed by Ross (1989). Two conditions were presented to children: standard place-value partitioning and nonstandard place-value partitioning. In the standard place-value condition, the tens place of the specified digit was represented by unit squares grouped together in tens, and the ones place was represented by individual unit squares. In the nonstandard condition, partitioning was the same with the exception that one of the groups of 10 was separated into 10 individual unit squares. Examples of standard and nonstandard partitioning arrangements are shown in Figure 1.

The experimenter first showed the child a card with the number 43 printed on it along with a picture of 43 squares in a standard place-value partitioning arrangement (i.e., four groups of 10 unit squares and 3 indi-

Table 3
*Approximate Arithmetic Problems*

| Arithmetic combination | Choices presented to children | |
|---|---|---|
| 1. 4 + 2 | **5** | 12 |
| 2. 3 + 6 | 18 | **10** |
| 3. 9 + 8 | **20** | 30 |
| 4. 7 + 9 | **18** | 10 |
| 5. 4 + 9 | 19 | **12** |
| 6. 5 + 6 | **10** | 16 |
| 7. 8 + 7 | 26 | **17** |
| 8. 3 + 8 | 17 | **10** |
| 9. 15 + 35 | **48** | 28 |
| 10. 42 + 17 | 90 | **60** |
| 11. 6 − 4 | 7 | **3** |
| 12. 16 − 7 | 4 | **8** |
| 13. 9 − 3 | **7** | 2 |
| 14. 13 − 4 | **8** | 2 |
| 15. 17 − 9 | **10** | 15 |
| 16. 11 − 5 | 9 | **5** |
| 17. 15 − 8 | 2 | **9** |
| 18. 11 − 8 | **2** | 10 |
| 19. 40 − 30 | **11** | 31 |
| 20. 50 − 9 | 20 | **40** |

*Note.* Values in bold represent the correct answer.

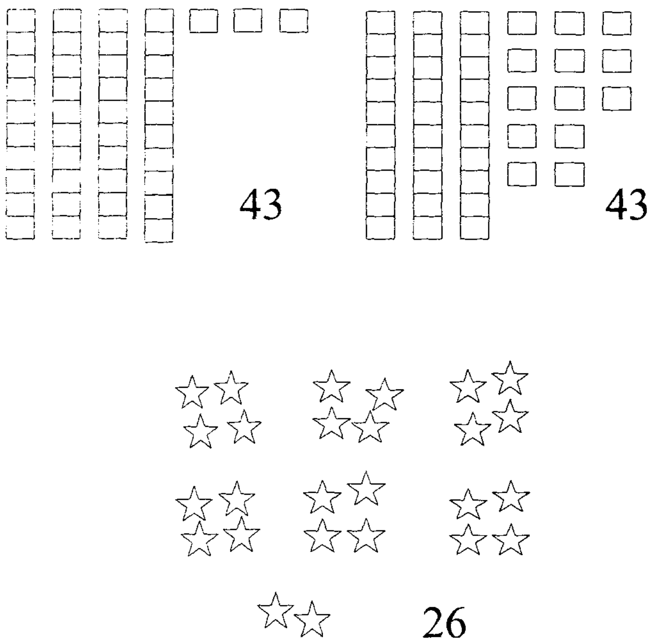


*Figure 1.* Standard and nonstandard partitioning arrangements. The first arrangement is standard and the second and third are nonstandard.

vidual unit squares). The experimenter said to the child, "There are 43 squares on the paper." The experimenter drew a circle around the 3 and said to the child, "Draw a circle around the squares that this part of the number 43 stands for." (The correct answer required the child to circle 3 squares.) The experimenter then circled the 4 and said to the child, "Draw a circle around the squares that this part of the number 43 stands for." (The correct answer required the child to circle 40 squares.) A second card with the number 43 printed on it and a corresponding picture of 43 squares was shown to the child. This time the squares were shown in a nonstandard partitioning arrangement (i.e., three groups of 10 unit squares and 13 individual unit squares). The experimenter then followed the procedure described for the standard partitioning item. The standard and nonstandard partitioning activities were repeated with the number 52.

In the final activity (also nonstandard partitioning), the child was shown a card with the number 26 printed on it along with a picture of 26 stars (see Figure 1). The stars were arranged in six groups of 4 stars and one group of 2 stars. The experimenter said to the child, "There are 26 stars on the paper." The experimenter drew a circle around the 6 and said to the child, "Draw a circle around the stars that this part of the number 26 stands for." (The correct answer required the child to circle 6 stars.) The experimenter then circled the 2 and said to the child, "Draw a circle around the stars that this part of the number 26 stands for." (The correct answer required the child to circle 20 stars.)

*Calculation principles.* Each child was asked to solve six pairs of problems in which the given answer to the first of the pair could be used to solve the second (Baroody, 1999; Russell & Ginsburg, 1984). Two items were given to assess understanding of (a) the commutativity principle, that the order of the addends does not affect the sum (i.e., 47 + 86 = 133, so 86 + 47 = ?, and 94 + 68 = 162, so 68 + 94 = ?); (b) the inversion principle, that subtraction is the inverse of addition (i.e., 27 + 69 = 96, so 96 − 69 = ?, and 36 + 98 = 134, so 134 − 36 = ?); and (c) the doubles plus one pattern (i.e., 37 + 37 = 74, so 37 + 38 = ?, and 64 + 64 = 128, so 65 + 64 = ?). The problems were presented to children both orally and visually (in a horizontal format). Children were told to give an oral response as soon as they knew the answer to the problem, but to respond quickly. To prevent children from calculating, a 5-s time limit was imposed. If a child did not respond within 5 s, the problem was marked "no answer" and scored as incorrect. We used two-digit numbers so children could not get answers simply by retrieving facts quickly.

*Forced retrieval of number facts.* This task is adapted from Jordan and Montani (1997) and requires children to retrieve answers to number facts quickly. Four addition and four subtraction problems were presented to each child (i.e., 4 + 2; 9 + 4; 7 + 9; 3 + 8, 6 − 4; 13 − 9; 16 − 7; and 11 − 8). As the problems were read to the children, they also were presented visually in a horizontal format. The experimenter read the addition problems as "How much is A plus B?" and the subtraction problems as "How much is A minus B?" Children were told to give an answer right away or to tell the experimenter that they would need more time to figure out the problem. Immediately after reading the problem, the experimenter began timing the child. If the child did not respond within 3 s, the experimenter marked "no answer" and the problem was scored as incorrect.

*Written computation.* Children were presented with eight two- and three-digit computation problems in an untimed written format. Four of the problems involved addition (45 + 23; 38 + 29; 624 + 312; 475 + 189) and four involved subtraction (67 − 31; 42 − 27; 849 − 524; 701 − 397). For both addition and subtraction problems, regrouping was necessary on half. The problems were printed vertically on two sheets of paper, with addition problems presented first. The eight problems were scored as being either right or wrong.

## Results

Statistical procedures for analyzing group differences included multivariate analysis of variance (MANOVA), univariate analysis of variance (ANOVA), and post hoc Tukey tests ($p < .05$).

### *Accuracy Data on the Mathematics Tasks*

The mean scores (based on the total number correct) for all of the mathematics tasks, broken down by achievement group, are presented in Table 4. Correlations among the tasks were positive and significant, with a range between .21 and .45.

There was a significant effect of achievement group on the math tasks, multivariate $F(21, 575) = 6.27$, $p < .0001$, $\eta^2 = .18$, power = 1.0. There was also a significant achievement group effect for each task: exact calculation of arithmetic combinations, $F(3, 206) = 22.05$, $p < .0001$, $\eta^2 = .24$; story problems, $F(3, 206) = 22.35$, $p < .0001$, $\eta^2 = .25$; approximate arithmetic, $F(3, 206) = 6.11$, $p < .001$, $\eta^2 = .08$; place value, $F(3, 206) = 11.09$, $p < .0001$, $\eta^2 = .14$; calculation principles, $F(3, 206) = 14.00$, $p < .0001$, $\eta^2 = .17$; forced retrieval of number facts, $F(3, 206) = 21.43$, $p < .0001$, $\eta^2 = .24$; and written computation, $F(3, 206) = 9.63$, $p < .0001$, $\eta^2 = .12$. The power values associated with the aforementioned statistical tests were at or near 1.0 for all analyses (1.0 for arithmetic combinations, story problems, calculation principles, place value, and forced retrieval and .96 for approximate arithmetic).

To understand the achievement group effects, we performed post hoc comparisons for each analysis (see Table 4). On exact calculation of arithmetic combinations, the MD/RD group performed worse than the MD-only group, the RD-only group, and the NA group; the MD-only group performed worse than the NA group. The advantage of the MD-only group over the MD/RD

Table 4
*Mean Scores on the Mathematics Tasks by Achievement Group*

| Achievement group | Exact calculation of arithmetic combinations ($n = 8$) | Story problems ($n = 10$) | Approximate arithmetic ($n = 20$) | Place value ($n = 12$) | Calculation principles ($n = 6$) | Forced retrieval of number facts ($n = 8$) | Written computation ($n = 8$) |
|---|---|---|---|---|---|---|---|
| MD-only | $5.64_{b}$ (1.98) | $4.72_{b}$ (1.81) | $12.75_{b}$ (2.34) | $4.43_{b}$ (1.74) | $2.58_{b,c,d}$ (1.71) | $1.64_{b}$ (1.29) | $2.09_{b,c,d}$ (1.83) |
| MD/RD | $4.37_{c}$ (1.95) | $3.62_{c}$ (1.76) | $12.73_{b}$ (2.77) | $4.21_{b}$ (1.83) | $1.79_{c,d}$ (1.55) | $1.38_{b}$ (1.39) | $1.65_{c,d}$ (1.61) |
| RD-only | $6.36_{a,b}$ (1.47) | $5.66_{a}$ (1.89) | $14.12_{a}$ (2.69) | $5.24_{b}$ (2.29) | $3.24_{a,b}$ (1.61) | $2.80_{a}$ (1.50) | $2.54_{a,b}$ (1.95) |
| NA | $6.85_{a}$ (1.16) | $6.42_{a}$ (1.99) | $14.36_{a}$ (2.45) | $6.44_{a}$ (2.82) | $3.71_{a}$ (1.61) | $3.38_{a}$ (1.75) | $3.38_{a}$ (1.57) |

*Note.* Standard deviations are shown in parentheses. Means in the same column that do not share at least one subscript differ significantly at $p < .05$ in the Tukey honestly significant difference comparison. MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.

group on exact calculation of arithmetic combinations is in keeping with our prediction. A similar performance pattern was found on story problems, where the MD/RD group performed worse than the MD-only group, the RD-only group, and the NA group; the MD-only group performed worse than the RD-only group and the NA group.

Children in the MD/RD and MD-only groups did not differ significantly from each other on the remaining mathematics tasks. We hypothesized that this would be the case on approximate arithmetic and on forced retrieval of number facts but not on the other tasks. On approximate arithmetic, the MD-only and MD/RD groups performed worse than the RD-only group and the NA group. On place value, the NA group performed better than the RD-only group, the MD-only group, and the MD/RD group. On calculation principles, the MD-only group and the MD/RD group performed worse than the NA group, and the MD/RD group performed worse than the RD-only group. On forced retrieval of number facts, the MD/RD group and the MD-only group performed worse than the RD-only group and the NA group. The insignificant difference between the RD-only and NA groups was not consistent with our expectation that RD-only children show subtle deficits in rapid fact retrieval. Finally, on written computation, the MD/RD group performed worse than the NA group and the RD-only group, but the MD-only group only performed worse than the NA group.

In summary, children in the MD-only and MD/RD groups performed worse than children in the NA group on all of the mathematics tasks. However, children in the MD-only group outperformed children in the MD/RD group on exact calculation of arithmetic combinations and on story problems. Children in the RD-only group performed about as well as children in the NA group on all of the tasks, with the exception of place value. The RD-only group performed better than both the MD-only and the MD/RD groups on story problems, approximate arithmetic, and forced retrieval and better than the MD/RD group on exact arithmetic combinations, calculation principles, and written computation.

### *Strategy and Response Time Data on Exact Calculation of Arithmetic Combinations and on Story Problems*

The mean response times and mean number of trials in which a strategy was used (along with percentage of trials in which a strategy produced a correct answer) are presented in Table 5 for exact calculation of arithmetic combinations and Table 6 for story problems.

*Exact calculation of arithmetic combinations.* To examine calculation efficiency, we first analyzed children's response times across all items. There was a significant effect of achievement group, $F(3, 206) = 11.07$, $p < .0001$, $\eta^2 = .14$, power $= .99$. Children in the NA group were significantly faster than children in each of the other three groups and children in the MD/RD group were significantly slower.

One-way ANOVAs were run on the mean number of trials in which each strategy was used. Achievement group effects were found for finger counting, $F(3, 206) = 3.49$, $p < .02$, $\eta^2 = .05$,

Table 5
*Mean Response Times and Mean Number of Trials in Which a Strategy Was Used Along With Percentage of Trials in Which a Strategy Produced a Correct Answer for Exact Calculation of Arithmetic Combinations*

| Achievement group | Response time | Automatic retrieval: No. of trials | Automatic retrieval: % correct | Delayed retrieval: No. of trials | Delayed retrieval: % correct | Derived fact: No. of trials | Derived fact: % correct | Verbal counting: No. of trials | Verbal counting: % correct | Finger counting: No. of trials | Finger counting: % correct |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MD-only | $11.37_{b}$ (5.48) | $0.79_{a}$ (1.04) | 84 | $0.77_{a}$ (1.34) | 39 | $0.49_{b}$ (1.17) | 100 | $1.23_{b}$ (1.73) | 52 | $4.62_{a,b}$ (2.73) | 77 |
| MD/RD | $14.35_{c}$ (7.60) | $0.58_{a}$ (1.33) | 43 | $0.71_{a}$ (1.26) | 63 | $0.42_{b}$ (1.35) | 88 | $0.46_{a}$ (0.80) | 46 | $5.63_{b}$ (2.45) | 57 |
| RD-only | $10.96_{b}$ (5.17) | $0.94_{a}$ (1.24) | 91 | $0.48_{a}$ (0.79) | 44 | $1.04_{a,b}$ (1.50) | 85 | $0.92_{a,b}$ (1.41) | 80 | $4.46_{a,b}$ (2.69) | 79 |
| NA | $7.85_{a}$ (4.78) | $1.16_{a}$ (1.38) | 91 | $0.55_{a}$ (1.14) | 82 | $1.33_{a}$ (1.87) | 98 | $0.85_{a,b}$ (1.11) | 81 | $4.04_{a}$ (2.66) | 83 |

*Note.* $n = 8$. Standard deviations are shown in parentheses. Means in the same column that do not share at least one subscript differ significantly at $p < .05$ in the Tukey honestly significant difference comparison. MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.

Table 6
*Mean Response Times and Mean Number of Trials in Which a Strategy Was Used Along With Percentage of Trials in Which a Strategy Produced a Correct Answer for Story Problems*

| Achievement group | Response time | Automatic retrieval: No. of trials | Automatic retrieval: % correct | Delayed retrieval: No. of trials | Delayed retrieval: % correct | Verbal counting: No. of trials | Verbal counting: % correct | Finger–penny counting: No. of trials | Finger–penny counting: % correct |
|---|---|---|---|---|---|---|---|---|---|
| MD-only | $18.96_{b}$ (9.23) | $0.92_{b}$ (1.49) | 47 | $1.85_{a,b}$ (2.02) | 35 | $0.19_{b,c,d}$ (0.56) | 58 | $6.87_{b}$ (3.06) | 51 |
| MD/RD | $16.26_{a,b}$ (9.08) | $1.52_{a,b}$ (2.22) | 33 | $1.25_{b}$ (1.61) | 22 | $0.08_{c,d}$ (0.27) | 75 | $6.98_{b}$ (3.15) | 39 |
| RD-only | $13.21_{a}$ (7.59) | $2.18_{a}$ (2.32) | 57 | $2.38_{a}$ (2.28) | 40 | $0.42_{a,b}$ (0.76) | 62 | $4.92_{a}$ (3.33) | 65 |
| NA | $13.47_{a}$ (9.27) | $2.36_{a}$ (2.53) | 63 | $2.00_{a,b}$ (1.98) | 60 | $0.60_{a}$ (0.93) | 73 | $4.87_{a}$ (3.34) | 67 |

*Note.* $n = 10$. Standard deviations are shown in parentheses. Means in the same column that do not share at least one subscript differ significantly at $p <$ .05 in the Tukey honestly significant difference comparison. MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.

power = .77; verbal counting, $F(3, 206) = 3.03$, $p < .03$, $\eta^2 = .04$, power = .71; and derived facts, $F(3, 206) = 4.54$, $p < .004$, $\eta^2 = .06$, power = .88. Children in the NA group used their fingers significantly less often than children in the MD/RD group, children in the MD-only group used verbal-counting strategies significantly more often than children in the MD/RD group, and children in the NA group used derived facts significantly more often than children in the MD-only group and the MD/RD group.

Although finger counting was the most frequently used strategy for all achievement groups, children in the MD/RD group used their fingers less accurately (57%) than children in the other three groups (near 80% for each group). In an examination of errors made by children in the MD/RD group on items where they used their fingers, these children made numerous counting mistakes (e.g., $15 - 8 = 6$; $8 + 7 = 16$). Accuracy with automatic retrieval was also relatively low for the MD/RD group (43%) compared with the other three groups (>80%).

*Story problems.* Children's response times on story problems varied with achievement group, $F(3, 206) = 4.91$, $p < .003$, $\eta^2 = .07$, power = .91. Children in the NA and the RD-only groups responded more quickly than children in the MD-only group.

One-way ANOVAs were run on the mean number of trials in which a particular strategy was used. There were significant achievement group effects for automatic retrieval, $F(3, 206) = 4.85$, $p < .003$, $\eta^2 = .07$, power = .90; delayed retrieval, $F(3, 206) = 2.87$, $p < .04$, $\eta^2 = .04$, power = .68; finger–penny counting, $F(3, 206) = 6.95$, $p < .0001$, $\eta^2 = .09$, power = .98; and verbal counting $F(3, 206) = 6.35$, $p < .0001$, $\eta^2 = .09$, power = .97. The MD-only group used automatic retrieval significantly less often than the NA and RD-only groups. The MD/RD group used delayed retrieval significantly less often than the RD-only group. The MD-only and the MD/RD groups used finger–penny counting significantly more often than the RD-only and the NA groups. The MD/RD group used verbal counting significantly less often than the RD-only and the NA groups, and the MD-only group used verbal counting less often than the NA group.

Finger–penny counting was the most frequently used strategy for all achievement groups on story problems. However, children in the MD/RD group used physical referents less accurately (39%) than children in the other three groups (>50%).

In an examination of children's errors on story problems, MD/RD children, in particular, attempted to add the two terms on the majority of story problems, even though only 3 of the 10 story problems required an addition operation (i.e., "Emily has 3 pennies. John has 6 pennies. How many pennies do they have altogether?"; "Janet has 3 pennies. Andy has 5 more pennies than Janet. How many pennies does Andy have?"; and "Anna has 7 pennies. She has 2 pennies less than Larry. How many pennies does Larry have?"). Supporting this observation, there were no achievement group effects on story problems requiring addition operations (means were about 50% correct for all achievement groups). On the remaining 7 problems (which required children to subtract), there was a significant achievement group effect, $F(3, 206) = 25.73$, $p < .0001$, $\eta^2 = .27$, power = 1.0. The MD/RD group performed significantly worse than children in the other three groups, and the MD-only group performed worse than the RD-only group and the NA group. The only subtraction problem that the majority of MD/RD children were able to solve was the simple change problem with an unknown result (i.e., "Nina has 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now?").

We also analyzed the mean distance of children's responses from the correct answer. Errors that are closer to the correct answer reflect more understanding of the problem (Levine et al., 1992). Consistent with the number correct data, the mean distance from the correct answer was 3.02 ($SD = 1.4$) for the MD/RD group, 2.3 ($SD = 1.4$) for the MD-only group, 1.6 ($SD = .96$) for the RD-only group, and 1.2 ($SD = .77$) for the NA group. There was a significant effect of achievement group, $F(3, 206) = 23.79$, $p < .0001$, $\eta^2 = .26$, power = 1.0. On post hoc comparisons, the MD/RD group's answers were significantly farther from the correct answer than those of the other three groups; the MD-only group's answers were significantly farther away from the correct answer than the RD-only and the NA groups' answers.

### Item Analyses on Place Value

To examine children's understanding of multidigit numbers, we analyzed performance on individual place-value items for each activity. The percentages of children who solved place-value items correctly, by achievement group, are displayed in Table 7.

Most second-grade children were able to count and identify numbers (with the exception of children in the MD/RD group who had trouble identifying a three-digit number). Consistent with our

Table 7
*Percentages of Children, by Achievement Group, Who Solved Place-Value Items Correctly*

| Achievement group | Counting: Count 16 chips | Number identification: 16 | Number identification: 37 | Number identification: 415 | Positional knowledge: 37 | Positional knowledge: 415 | Digit correspondence: 16 (show what 6 and 10 mean with chips) | Digit correspondence: 43 standard | Digit correspondence: 43 nonstandard | Digit correspondence: 52 standard | Digit correspondence: 52 nonstandard | Digit correspondence: 26 nonstandard |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MD-only | 85 | 98 | 98 | 68 | 25 | 11 | 9 | 19 | 6 | 17 | 6 | 2 |
| MD/RD | 90 | 100 | 98 | 35 | 38 | 13 | 6 | 13 | 2 | 17 | 4 | 4 |
| RD-only | 92 | 100 | 100 | 76 | 32 | 24 | 12 | 24 | 10 | 30 | 14 | 10 |
| NA | 95 | 98 | 100 | 93 | 36 | 27 | 29 | 44 | 27 | 45 | 33 | 16 |

*Note.* MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.

predictions and the previous literature, however, the majority of second graders had trouble with positional knowledge and digit correspondence activities, which were designed specifically to get at children's understanding of place-value and base-10 concepts. An ANOVA on digit correspondence scores ($n = 6$) alone, by achievement group, $F(3, 206) = 9.02$, $p < .0001$, $\eta^2 = .12$, power = 1.0, shows the same pattern of findings as the previously reported ANOVA on the place-value scores overall (i.e., the NA group performed significantly better than the RD-only, MD-only, and MD/RD groups. Although children in the NA-group missed a lot of place-value understanding items, they still have an early advantage over children in each of the three other achievement groups.

### *Item Analyses on Calculation Principle*

Table 8 examines children's scores, broken down by principle (i.e., commutativity, inversion, and doubles plus one). Overall, children were most successful with the commutativity principle, second most successful with the doubles plus one pattern, and least successful with the inversion principle. For each principle, the NA group performed best and the MD/RD group performed worst. Because only two items were used to assess each principle ($n = 2$), we did not perform statistical analyses on the data.

### *Operation and Error Analyses on Forced Retrieval of Number Facts*

Because forced retrieval of number facts was difficult for all children and there was a floor effect on subtraction problems, we performed an ANOVA on forced retrieval of addition problems only. The mean scores (out of a total of 4) were 1.3 ($SD = 0.98$) for the MD-only group, 1.1 ($SD = 1.1$) for the MD/RD group, 1.9 ($SD = 1.0$) for the RD-only group, and 2.4 ($SD = 0.91$) for the NA group. There was a significant effect of achievement group, $F(3, 206) = 18.11, p < .0001, \eta^2 = .21$, power = 1.0. Both MD groups performed significantly worse than the RD-only and NA groups. However, the RD-only group performed significantly worse than the NA group on forced retrieval addition problems, a finding that is different from the combined addition and subtraction analysis, where no differences were found between the two achievement groups. This result is in keeping with the prediction of relative weaknesses in rapid retrieval among children with RD-only.

To determine whether children's errors on forced retrieval of number facts reflect intrusions of related associations (Geary et al., 2000), we examined our data for the occurrence of "counting-string associate" errors on number facts involving addition (Siegler & Shrager, 1984). The counting-string associate is the number that is one higher than an addend (e.g., the counting-string associates for 9 + 4 would be 10 and 5, respectively). Although counting-string associate errors were observed more frequently in children with MD than in children without MD, they were uncommon for children in all achievement groups. The percentage of errors that were counting-string associates was 8 for the MD-only group, 8 for the MD/RD group, 2 for the RD-only group, and 2 for the NA group.

Table 8
*Mean Number of Calculation Principles, by Achievement Group, That Were Solved Correctly*

| Achievement group | Commutativity ($n = 2$) | Doubles + one ($n = 2$) | Inversion ($n = 2$) |
|---|---|---|---|
| MD-only | 1.17 (0.91) | 0.79 (0.86) | 0.62 (0.81) |
| MD/RD | 1.08 (0.95) | 0.38 (0.69) | 0.33 (0.62) |
| RD-only | 1.56 (0.70) | 1.04 (0.86) | 0.64 (0.80) |
| NA | 1.67 (0.64) | 1.22 (0.90) | 0.82 (0.82) |

*Note.* Standard deviations are shown in parentheses. MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.

### *Regrouping Versus Nonregrouping on Written Computation*

Table 9 contains children's performance on written computation, broken down by complexity (regrouping vs. nonregrouping). On nonregrouping problems, there was a significant effect of achievement group, $F(3, 206) = 7.47$, $p < .0001$, $\eta^2 = .10$, power = .99. The MD/RD and the MD-only groups performed significantly worse than the NA group but not the RD-only group. Because of floor effects, we did not perform statistical analyses on regrouping problems.

## Discussion

We examined mathematical competencies in second-grade children with different patterns of achievement in mathematics and reading.

Table 9
*Mean Number of Written-Computation Problems, by Achievement Group and Problem Type, That Were Solved Correctly*

| Achievement group | Nonregrouping problems ($n = 4$) | Regrouping problems ($n = 4$) |
|---|---|---|
| MD-only | $1.96_b$ (1.75) | 0.13 (0.34) |
| MD/RD | $1.62_b$ (1.59) | 0.04 (0.19) |
| RD-only | $2.22_{a,b}$ (1.72) | 0.32 (0.55) |
| NA | $3.00_a$ (1.23) | 0.36 (0.68) |

*Note.* Standard deviations are shown in parentheses. Means that do not share at least one subscript differ significantly at $p < .01$ in the Tukey honestly significant difference comparison. MD = mathematics difficulties; RD = reading difficulties; NA = normal achievement.

### Basic Calculation

We found that second-grade children with MD-only had an advantage over children with MD/RD on exact calculation of arithmetic combinations, even though both groups performed worse than NA children. RD-only children did not differ from MD-only or NA children, but they performed significantly better than MD/RD children. Finger counting was the most common calculation strategy for all children on arithmetic combinations. However, children with MD/RD relied on their fingers more often than NA children. Although accuracy of finger counting was high for the MD-only, RD-only, and NA groups, it was substantially lower for the MD/RD group. Consistent with the findings of Geary (1990), children with MD/RD often over- or undercounted by 1 with their fingers. Children with MD/RD used counting procedures less skillfully than children with MD-only or children without MD (see also Geary et al., 1999, 2000).

The MD-only group did not have a significant advantage over the MD/RD group on the forced retrieval of number facts task, where children were not allowed to use counting or other backup calculation strategies. Both the MD-only and the MD/RD groups performed worse than the non-MD groups. Deficits in rapid retrieval of number facts are present early in children with both general and specific mathematics difficulties (see also Geary et al., 1999). Fact retrieval deficits among children with MD persist throughout elementary school, even when intervention is provided (Jordan & Montani, 1997; Ostad, 1997, 1999).

Were there reliable differences between RD-only and NA children on forced retrieval of number facts? When we examined children's performance on addition problems only (because there was a floor effect on subtraction problems), the NA group had a small but significant advantage over the RD-only group. Geary et al. (2000) found a similar effect for children with RD on a "retrieval only" addition task. Number fact retrieval and word reading may share a common cognitive factor associated with representing and retrieving information from phonetic and semantic memory (Geary, 1993; Rasanen & Ahonen, 1995), but further explanation is needed as to why children with MD-only (who do not have associated reading difficulties) also show serious fact retrieval deficiencies. Researchers have conjectured that some retrieval deficits can be explained by inefficient inhibition of irrelevant associations (Barrouillet, Fayol, & Lathuliere, 1997; Geary et al., 2000). On addition problems, for example, inefficient inhibition of irrelevant associations can be indexed by a high percentage of retrieval errors that were counting-string associates of an addend (e.g., stating 10 or 5 for 9 + 4). Geary et al. (2000) found a relatively high percentage of retrieval errors that were counting-string associates in children with MD-only (17%), children with MD/RD (29%) and children with RD-only (21%), but not in NA children (5%). In the present investigation, however, we found a much lower percentage of counting-string associates on retrieval errors for children in the three difficulty groups (8% for children with MD-only or with MD/RD and 2% for children with RD-only). Thus, the extent to which the inefficient inhibition of irrelevant associations hypothesis explains fact retrieval deficits is not clear and warrants further investigation.

The performance of children with MD-only and children with MD/RD was not differentiated on the calculation principles task. Both groups of children with MD performed worse than NA children, and children with MD/RD also performed worse than children with RD-only. Difficulties with calculation principles or patterns among children in both MD groups also is reflected in the relatively low number of trials in which they used derived fact strategies on the exact calculation of number facts task. Children with MD may have a tenuous grasp of relationships between and within arithmetic operations. It should be noted, however, that for children in all achievement groups the commutativity principle items were the easiest and the inversion principle items the most difficult. There was generally weak performance on inversion principle items (e.g., 27 + 69 = 96, so 96 − 69 = 27)—relationships between addition and subtraction are not well established in second graders, which may be a result of instructional approaches (e.g., subtraction not being taught in relation to addition).

### Approximate Arithmetic

On the approximate arithmetic task, children with MD performed significantly worse than children without MD. These findings were present irrespective of reading ability. The ability to make arithmetical approximations may be a core deficit for children with MD. Relatively strong reading (and, by association, language) skills in children with low mathematics performance do not seem to result in better performance. The ability to perform approximate arithmetic tasks seems to be independent of language (Dehaene et al., 1999). On the basis of data obtained through functional brain-imaging techniques designed to examine underlying neural circuitry (i.e., fMRI), Dehaene et al. (1999) posit that "approximate arithmetic involves a representation of numerical quantities analogous to a spatial number line, which relies on visuo-spatial circuits of the dorsal parietal pathway" (of the brain; p. 972). It remains to be seen whether difficulties of children with MD on approximate arithmetic are related to an underlying spatial deficit. Moreover, the effect of instruction on children's ability to form arithmetical approximations remains an open question.

### Problem Solving

Children with MD/RD showed particular weaknesses in solving orally presented story problems, relative to the performance of children in the other three achievement groups. This finding is in keeping with those of previous investigations (Jordan & Hanich, 2000; Jordan & Montani, 1997). As on exact calculation of arithmetic combinations, children in all achievement groups relied on counting physical referents (i.e., pennies or fingers) for solving story problems. However, children with MD-only and children with MD/RD counted referents more often than children without MD (i.e., NA and RD-only children). Children with MD/RD counted less accurately than children in the other achievement groups, which replicates our findings on the exact calculation of arithmetic combinations task.

Children in the MD/RD group had a disadvantage in solving story problems, as shown by the nature of their errors as well as by their performance level. Children with MD/RD used addition as a default strategy on complex problems more often than children in the other three achievement groups. For example, on the problem "Karen had some pennies. Then Matt gave her 4 more pennies. Now Karen has 6 pennies. How many pennies did she have to start with?" The MD/RD group frequently added the two numbers rather than subtracting them. In analyses on distance of errors from the correct answer (an index of understanding of the problem), the MD/RD group's errors were significantly farther from the correct answer than were the errors of the MD-only group.

### *Place Value and Written Computation*

On the place-value task, children in the NA group performed better than children in the other three achievement groups, and children in the RD-only group performed better than children in the MD/RD group. All children performed well on counting and number identification tasks (with the exception of three-digit number identification, which was problematic for the MD/RD group). Positional knowledge and digit correspondence tasks were hard for most second graders, although they performed better on standard than on nonstandard partitioning tasks related to digit correspondence. Overall, second graders had only partial understanding of place-value and base-10 concepts. However, NA children were further ahead in their understanding than children with learning difficulties, and children with MD/RD seemed to lag behind children with specific difficulties in mathematics or reading.

Finally, on written computation that did not require regrouping, children with both kinds of MD performed worse than NA children but not than children with RD-only. Written computation involving regrouping was problematic for all second graders. Longitudinal data are needed to examine children's developmental trajectory in place-value understanding and the relationship between place-value understanding and later written computational skill.

In conclusion, children with MD-only should be considered separately from children with MD/RD. In previous research, these subgroups were confounded, making it difficult to interpret the results. Children with MD-only may have a particular advantage over children with MD/RD on skills that may be acquired in a language-specific format or that can be mediated with language (i.e., exact calculation of arithmetic combinations and story problems) but not on tasks that rely on numerical magnitudes, visuospatial processing, and automaticity. Currently, we are studying children longitudinally to examine change in mathematical competencies and stability of MD between second and fourth grades.

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